Unit 3 – 4

Solve the following by completing the square. Derive the quadratic formula.

$$a \cdot x^2 + b \cdot x + c = 0$$

$-c \quad -c$ ← MOVE THE CONSTANT TO THE OTHER SIDE.

$$\frac{ax^2 + bx}{a} = \frac{-c}{a}$$ ← DIVIDE BOTH SIDES BY THE LEADING COEFFICIENT.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$ ← COMPLETE THE SQUARE BY ADDING $\frac{b^2}{4a^2}$ TO BOTH SIDES

$\frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a}$ → $\frac{b}{2a}$ (1/2 IT) → $\frac{b^2}{4a^2}$ (SQR IT)

$$= \frac{-c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{4a^2}$$ ← MUST HAVE A COMMON DENOMINATOR TO ADD FRACTIONS ON THE RIGHT.

$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$ ← FACTOR THE LEFT SIDE.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$ ← USE THE SQUARE ROOT METHOD TO "UNDO" THE SQUARED ON THE LEFT.

$$x + \frac{b}{2a} = \frac{\pm\sqrt{-4ac + b^2}}{2a}$$ ← ISOLATE THE VARIABLE x BY ADDING $-\frac{b}{2a}$ TO BOTH SIDES.

AND SIMPLIFY THE DENOMINATOR $\sqrt{4a^2} = 2a$

$-\frac{b}{2a} \quad -\frac{b}{2a}$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

Solve the following by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $x^2 + 10x - 19 = 0$

$a=1$ $b=10$ $c=-19$

$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-19)}}{2(1)} = \frac{-10 \pm \sqrt{100 + 76}}{2}$

$x = \frac{-10 \pm \sqrt{176}}{2} = \frac{-10 \pm 4\sqrt{11}}{2}$

$x = -5 \pm 2\sqrt{11}$ ← EXACT

$\sqrt{176} = 2\sqrt{88} = 4\sqrt{11}$

2. $x^2 - 12x + 2 = 0$

$a=1$ $b=-12$ $c=2$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(2)}}{2(1)}$

$x = \frac{12 \pm \sqrt{144 - 8}}{2} = \frac{12 \pm \sqrt{136}}{2}$

$x = \frac{12 \pm 2\sqrt{34}}{2} = 6 \pm \sqrt{34}$ ← EXACT

$\sqrt{136} = 2\sqrt{34}$

3. $x^2 + 15x - 8 = 0$

$a=1$ $b=15$ $c=-8$

$x = \frac{-(15) \pm \sqrt{(15)^2 - 4(1)(-8)}}{2(1)} = \frac{-15 \pm \sqrt{225 + 32}}{2}$

$x = \frac{-15 \pm \sqrt{257}}{2}$ ← EXACT

$\sqrt{257}$ DOESN'T SIMPLIFY

$x \approx -15.52$ or 0.52 ← APPROX.

4. $x^2 - 7x - 4 = 0$

$a=1$ $b=-7$ $c=-4$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-4)}}{2(1)}$

$x = \frac{7 \pm \sqrt{49 + 16}}{2} = \frac{7 \pm \sqrt{65}}{2}$

$x = \frac{7 \pm \sqrt{65}}{2}$ ← EXACT

$\sqrt{65}$ DOESN'T SIMPLIFY

$x \approx -0.53$ or 7.53 ← APPROXIMATE

5. $2x^2 - 8x - 14 = 0$

$a=2$ $b=-8$ $c=-14$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-14)}}{2(2)} = \frac{8 \pm \sqrt{64 + 112}}{4}$

$x = \frac{8 \pm \sqrt{176}}{4} = \frac{8 \pm 4\sqrt{11}}{4}$

$x = 2 \pm \sqrt{11}$ ← EXACT

$x \approx -1.32$ or 5.32

$\sqrt{176} = 4\sqrt{11}$

6. $3x^2 - 18x - 5 = 0$

$a=3$ $b=-18$ $c=-5$

$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(3)(-5)}}{2(3)}$

$x = \frac{18 \pm \sqrt{324 + 60}}{6} = \frac{18 \pm \sqrt{384}}{6}$

$x = \frac{18 \pm 8\sqrt{6}}{6} = \frac{9 \pm 4\sqrt{6}}{3}$ ← EXACT

$x \approx -0.27$ or 6.27 ← APPROX.

$\sqrt{384} = 8\sqrt{6}$

7. $2x^2 - 10x - 3 = 0$

$a=2$ $b=-10$ $c=-3$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-3)}}{2(2)} = \frac{10 \pm \sqrt{100 + 24}}{4}$

$x = \frac{10 \pm \sqrt{124}}{4} = \frac{10 \pm 2\sqrt{31}}{4}$

$x = \frac{5 \pm \sqrt{31}}{2}$ ← EXACT

$x \approx 5.28$ or -0.28 ← APPROX

$\sqrt{124} = 2\sqrt{31}$

8. $3x^2 - 21x - 5 = 0$

$a=3$ $b=-21$ $c=-5$

$x = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(3)(-5)}}{2(3)} =$

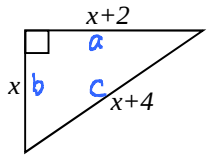
$x = \frac{21 \pm \sqrt{501}}{6}$ ← EXACT

$x \approx 7.23$ or -0.23 ← APPROXIMATE

$\sqrt{501}$ DOESN'T SIMPLIFY

Applications

1. Find the value of x that would make the diagram below accurate.



$(x)^2 + (x+2)^2 = (x+4)^2$

$x^2 + (x+2)(x+2) = (x+4)(x+4)$

$x^2 + x^2+2x+2x+4 = x^2+4x+4x+16$

$2x^2 + 4x + 4 = x^2 + 8x + 16$

$-x^2 \quad -8x \quad -16 \quad -x^2 \quad -8x \quad -16$

$x^2 - 4x - 12 = 0$

$a=1 \quad b=-4 \quad c=-12$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(-12)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16+48}}{2} = \frac{4 \pm \sqrt{64}}{2}$

$x = \frac{4+8}{2}$ or $\frac{4-8}{2}$

$x = 6$ or ~~-2~~ ← EXTRANEOUS

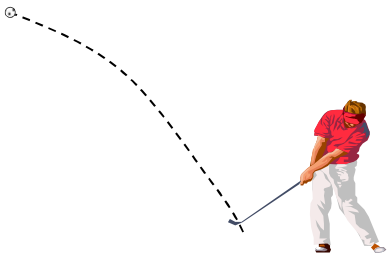
LEG 1 = $x = 6$

LEG 2 = $x+2 = 8$

HYP = $x+4 = 10$

2. A golf ball is hit with an initial vertical velocity of 80 fps

$$h = -16t^2 + 80t$$



a. How high is the ball after 2 seconds?

$h = -16(2)^2 + 80(2)$

$h = 96$ ft

```
-16(2)^2+80(2)
                96
```

b. How many seconds would it take the ball to hit the ground (the height would be h=0)?

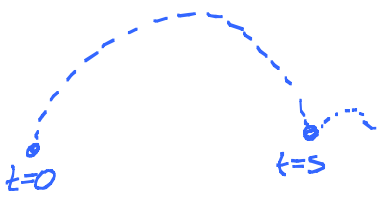
$0 = -16t^2 + 80t$

$a=-16 \quad b=80 \quad c=0$

$x = \frac{-(80) \pm \sqrt{(80)^2-4(-16)(0)}}{2(-16)}$

$x = \frac{-80 \pm \sqrt{6400}}{-32} = \frac{-80 \pm 80}{-32} = \frac{-80+80}{-32}$ or $\frac{-80-80}{-32}$

0 or 5



c. When will the ball reach 48 feet?

$48 = -16t^2 + 80t$

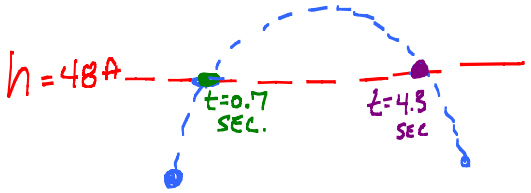
$-48 \quad\quad -48$

$0 = -16t^2 + 80t - 48$

$a=-16 \quad b=80 \quad c=-48$

```
16)(-48)))/(2(-1
6))
                .6972243623
(-80-√((80)²-4(-
16)(-48)))/(2(-1
6))
                4.302775638
```

$x = \frac{-(80) \pm \sqrt{(80)^2-4(-16)(-48)}}{2(-16)} \approx 0.70$ secs or 4.30 secs



d. What is the average RATE OF change from t = 1 to t = 2 seconds?

POINT #1 $t=1$ $(1, 64)$ $h = -16(1)^2 + 80(1) = 64$

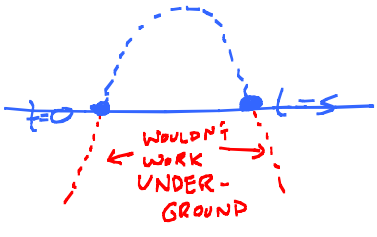
POINT #2 $t=2$ $(2, 96)$ $h = -16(2)^2 + 80(2) = 96$

SLOPE $(m) = \frac{h_2-h_1}{t_2-t_1} = \frac{96-64}{2-1} = \frac{32}{1} = \boxed{32 \text{ ft/sec}}$

e. For what values of t is the domain appropriate?

DOMAIN: $0 \le t \le 5$ ← SET NOTATION

$[0, 5]$ ← INTERVAL NOTATION



3. A baseball his hit with an initial vertical velocity of 121 fps an the ball was struck 1 foot above ground.

$$h = -16t^2 + 121t + 1$$

a. How high is the ball after 2 seconds?

$h = -16(2)^2 + 121(2) + 1$

$h = 179$ ft

-16(2)²+121(2)+1
179

b. How many seconds would it take the ball to hit the ground (the height would be *h* = 0)?

$0 = -16t^2 + 121t + 1$

$a=-16$ $b=121$ $c=1$

$t = \dfrac{-(121) \pm \sqrt{(121)^2 - 4(-16)(1)}}{2(-16)}$

$t \approx$ ~~-.008~~ EXTRANEOUS OR 7.571 SECONDS

(-16)(1)))/(2(-16))
-.0082554509
(-121-√((121)²-4(-16)(1)))/(2(-16))
7.570755451

c. When will the ball reach 30 *feet*?

$30 = -16t^2 + 121t + 1$

$-30 \quad\quad -30$

$0 = -16t^2 + 121t - 29$

$a=-16$ $b=121$ $c=-29$

$t = \dfrac{-(121) \pm \sqrt{(121)^2 - 4(-16)(-29)}}{2(-16)}$

$t \approx 0.248$ SECS OR 7.315 SECS

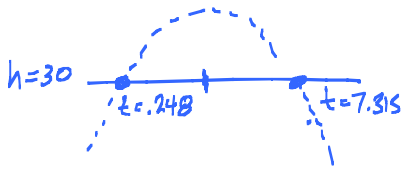
(-16)(-29)))/(2(-16))
.2477883032
(-121-√((121)²-4(-16)(-29)))/(2(-16))
7.314711697

d. When does the ball reach a maximum height?

$\dfrac{0.248 + 7.315}{2}$

(0.248+7.315)/2
3.7815
(-.008+7.57)/2
3.781

$t \approx 3.78$ SECS



(96.94-24.56)/(.9-.2)
103.4

e. What is the average vertical velocity from t = 0.2 to t = 0.9 seconds?

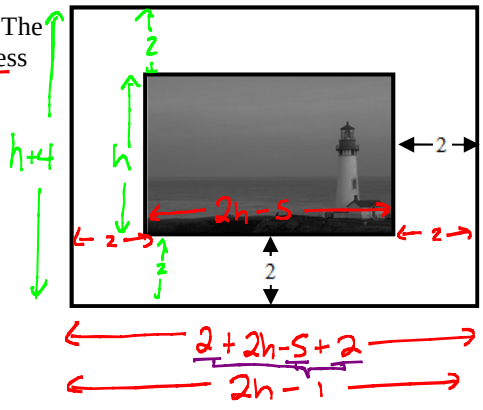
POINT #1 (0.2, 24.56) $h = -16(0.2)^2 + 121(0.2) + 1 = 24.56$

POINT #2 (0.9, 96.94) $h = -16(.9)^2 + 121(.9) + 1 = 96.94$

-16(.2)²+121(.2)+1
24.56
-16(.9)²+121(.9)+1
96.94

$M = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{96.94 - 24.56}{.9 - .2} = 103.4$ ft/sec

4. A person at a framing store is making a frame mat to go around a picture. The mat is a uniform 2 inches around on each side. The picture's width is 5 less than twice the picture's height. The entire area of the frame with picture included is 221 square inches. What are the dimensions of the picture?



$A = h \cdot w$

$221 = (h+4)(2h-1)$

$221 = 2h^2 - 1h + 8h - 4$

$-221 \quad\quad -221$

$0 = 2h^2 + 7h - 225$

$a=2$ $b=7$ $c=-225$

$h = \dfrac{-(7) \pm \sqrt{(7)^2 - 4(2)(-225)}}{2(2)}$

$= 9$ INCH OR ~~-12.5 INCH~~ EXTRANEOUS

(-7+√((7)²-4(2)(-225)))/(2(2))
9
(-7-√((7)²-4(2)(-225)))/(2(2))
-12.5

HEIGHT = $h = 9$ INCH

WIDTH = $2h - 5 =$
$= 2(9) - 5$
$= 18 - 5 = 13$ INCH

PICTURE DIM: 9 IN X 13 IN.