Unit 3 – 6

Name:

1. Consider the function: $f(x) = x^2 + x - 6$ ← STANDARD FORM

$f(x) = (x-2)(x+3)$ ← INTERCEPT FORM

a. What are the zeros of the function (using factoring)?

$0 = x^2 + x - 6$

$0 = (x-2)(x+3)$

$x-2=0$, $+2\ +2$, $x=2$; $x+3=0$, $-3\ -3$, $x=-3$

-6: $1, -6$; $-1, 6$; $2, -3$; $-2, 3$

THE ZEROS OCCUR AT $x=2$ AND $x=-3$

b. What is the axis of symmetry of the parabola?

AVERAGE THE 2 X-INTERCEPTS OR ZEROS OF f(x): $\frac{2+-3}{2} = \frac{-1}{2} = -0.5$

AXIS OR LINE OF SYMMETRY: $x = -0.5$

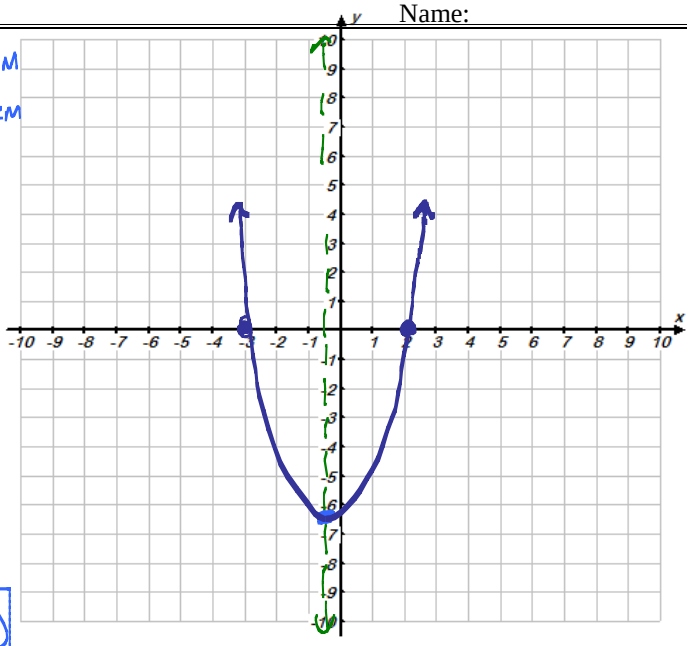
c. What is the vertex of the parabola (graph the parabola)?

$f(-0.5) = (-0.5)^2 + (-0.5) - 6$

$(-.5)^2+(-.5)-6$ → -6.25

$= -6.25$

VERTEX: $(-0.5, -6.25)$



2. Consider the function: $f(x) = -x^2 + 8x - 15$

a. What are the zeros of the function (using factoring)?

$f(x) = -1(x^2 - 8x + 15)$

$f(x) = -1(x-3)(x-5)$

$x-3=0$, $+3\ +3$, $x=3$; $x-5=0$, $+5\ +5$, $x=5$

15: $1, 15$; $-1, -15$; $3, 5$; $-3, -5$

b. What is the axis of symmetry of the parabola?

AVERAGE THE 2 X-INTERCEPTS OR ZEROS OF f(x): $\frac{3+5}{2} = \frac{8}{2} = 4$

AXIS OF SYM: $x = 4$

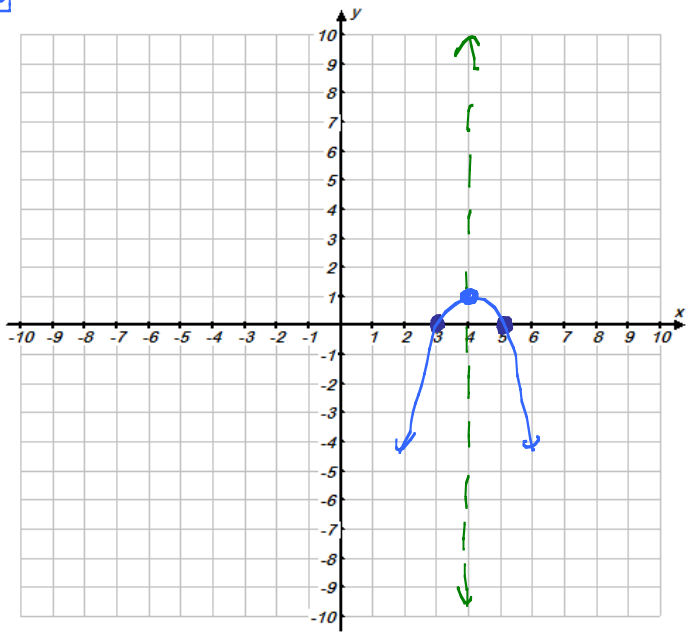
c. What is the vertex of the parabola (graph the parabola)?

$f(4) = -(4)^2 + 8(4) - 15 = 1$

$-(4)^2+8(4)-15$ → 1

$= 1$

$(4, 1)$



3. Consider the function: $f(x) = 2x^2 + x - 3$

a. What are the zeros of the function (using factoring)?

USE THE "AC" METHOD TO FACTOR B/C LEAD COEFF ≠ 1

$0 = 2x^2 + x - 3$

$0 = \frac{2x-2}{2}\left(\frac{2x+3}{1}\right)$

$0 = (x-1)(2x+3)$

-6: $1, -6$; $-1, 6$; $2, -3$; $-2, 3$

$x-1=0$, $+1\ +1$, $x=1$; $2x+3=0$, $-3\ -3$, $\frac{2x}{2} = \frac{-3}{2}$

X-INTERCEPTS / ZEROS: $x=1$ AND $x=-1.5$

b. What is the axis of symmetry of the parabola?

AVERAGE THE 2 X-INTERCEPTS OR ZEROS OF f(x): $\frac{-1.5+1}{2} = \frac{-0.5}{2} = -0.25$

AXIS OF SYM: $x = -0.25$

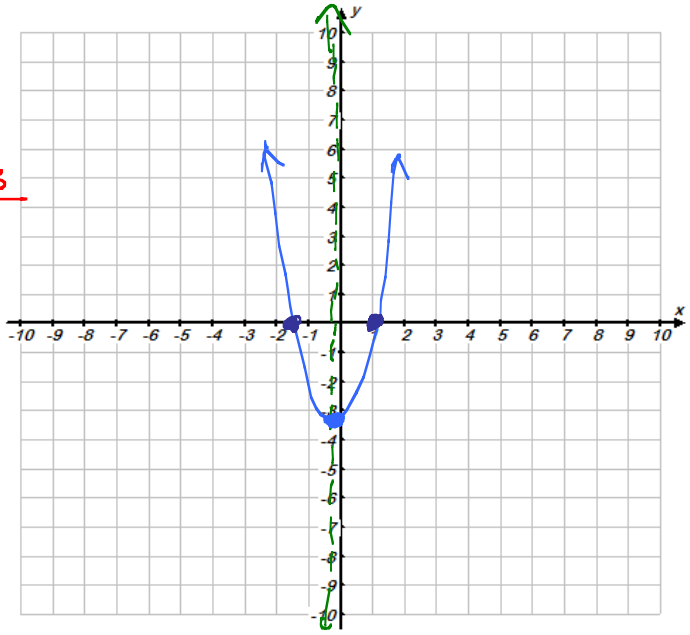
c. What is the vertex of the parabola (graph the parabola)?

$f(-0.25) = 2(-0.25)^2 + (-0.25) - 3$

$2(-.25)^2+(-.25)-3$ → -3.125

$= -3.125$

VERTEX: $(-0.25, -3.125)$



23. The expression $P = -x^2 + 70x - 600$ represents a company's profit for selling x items.

a. What are the break-even point(s) for selling x items (i.e. how many items sold yields a profit of 0?)

$0 = -x^2 + 70x - 600$

$0 = -1(x^2 - 70x + 600)$

$0 = -1(x-60)(x-10)$

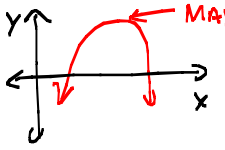
$x-60=0$ $\quad x-10=0$

600: $60, 10$; $-60, -10$

$x-60=0$, $+60\ +60$, $x=60$

$x-10=0$, $+10\ +10$, $x=10$

b. Is the vertex a minimum or maximum?



LEAD COEFFICIENT $= -1$

$P = -x^2 + 70x - 600$

SINCE THE LEADING COEFFICIENT IS NEGATIVE THE PARABOLA WILL OPEN DOWN WHICH MAKES THE VERTEX A MAXIMUM.

c. If the model is accurate, how many items should the company sell to maximize their profit and what is the maximum profit?

METHOD #1

AVG OF INTERCEPTS: $\frac{60+10}{2} = \frac{70}{2} = 35$

$P = -(35)^2 + 70(35) - 600$

$= 625$

$(35, 625)$

METHOD #2

$\frac{-b}{2a} = \frac{-70}{2(-1)} = \frac{-70}{-2} = 35$

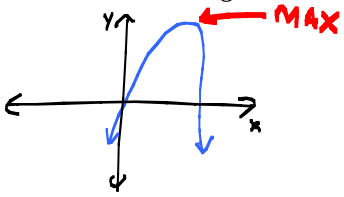
$P = -(35)^2 + 70(35) - 600$

$= 625$

$(35, 625)$

WHEN THE COMPANY SELLS 35 ITEMS THEY MAXIMIZE THEIR PROFITS TO $625.

24. The expression $h = -16t^2 + 400t + 5$ represents the height of a cannonball t seconds after it was fired. What is the maximum height of the cannon ball and how many seconds did it take to reach its maximum height?



PARABOLA MUST OPEN DOWN BECAUSE THE LEADING COEFFICIENT IS NEGATIVE AND CREATES A MAXIMUM AT THE VERTEX

-16(12.5)²+400(12.5)+5
2505

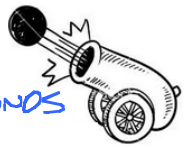
VERTEX: $\left(\frac{-b}{2a}, \right)$

$\left(\frac{-400}{2(-16)}, \right)$

$(12.5, 2505)$

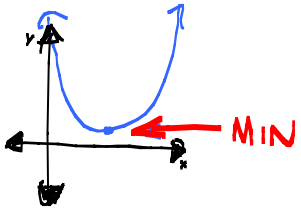
$h = -16(12.5)^2 + 400(12.5) + 5$

$h = 2505$



AFTER 12.5 SECONDS THE CANNON BALL REACHES A MAXIMUM HEIGHT OF 2505 ft

25. The expression $C = x^2 - 44x + 490$ represents the cost in $1000 of dollars per year that company must spend out of pocket on each employee for health insurance for x number of employees. How many employees should the company hire to minimize their cost of health insurance?



PARABOLA MUST OPEN UP BECAUSE THE LEADING COEFFICIENT IS POSITIVE AND CREATES A MINIMUM AT THE VERTEX

(22)²-44(22)+490
6

VERTEX: $\left(\frac{-b}{2a}, \right)$

$\left(\frac{44}{2(1)}, \right)$

$(22, 6)$

$C = (22)^2 - 44(22) + 490 = 6$

THE COMPANY SHOULD HIRE 22 EMPLOYEES TO MINIMIZE THEIR COST TO 6 THOUSAND DOLLARS.