Integrated Geometry Name KEY

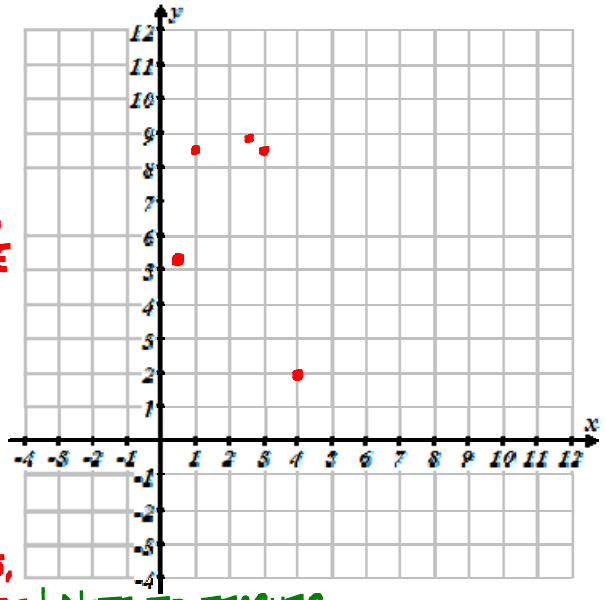
The businesses that are most likely to succeed are those that pay close attention to details and trends. 50% of all small businesses FAIL in the first year (Source: U.S. Small Business Administration). A business owner owned two "Bruce & Stirs" ice cream shops. Both business are in very similar locations and sell approximately the same amount of products. The owner was experimenting with different numbers of employees for each location and collected the following data for a few different summer months:

	Number of Employees, *(x)*	4	3	0.5	1	2.5
	Gross Profit *(in Thousands of Dollars)* *(y)*	2	8.5	5.2	8.6	8.9

1) What do you think is meant by 0.5 employees?
IT MIGHT SUGGEST A SINGLE PART-TIME EMPLOYEE THAT WORKS HALF OF A DAY.

2) How many total hours of work would have been worked by 0.5 employees? IF WE ASSUME 1 FULL-TIME EMPLOYEE WORKS 40 HOURS PER WEEK THEN 0.5 MIGHT SUGGEST A PART-TIME EMPLOYEE THAT WORKS 20 HOURS PER WEEK.

3) Plot the data on the graph at the right.



a. Describe the shape or pattern the data makes.
THE DATA MAKES AN UPSIDE DOWN U-SHAPE.
(IT INCREASES, SEEMS TO LEVEL OFF, AND THEN DECREASE)

b. Can you make a hypothesis as to why such a pattern might occur?
WITH TOO FEW EMPLOYEES, THERE MAY NOT BE ENOUGH HELP TO SERVE ALL OF THE CUSTOMERS AND BUSINESS IS LOST. WITH TOO MANY EMPLOYEES, ALL OF THE CUSTOMERS ARE SERVED BUT THEN THE OWNER HAS TO PAY MORE FOR ALL OF THE HELP.

NOTE TO TEACHER:
STUDENTS MAY NEED ADDITIONAL ASSISTANCE WITH PART 3a. STUDENTS DON'T USUALLY TAKE ECONOMICS UNTIL THE 12TH GRADE. PREPARE FOR BRIEF DISCUSSIONS ON TERMS SUCH AS "GROSS PROFIT".

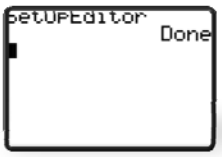
4) Using the data to find a quadratic model that represents the data using a quadratic regression.
The following are the directions for the TI-83/84:

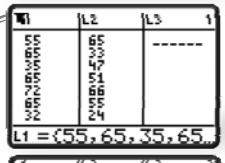
A) Under the Stat menu, press STAT 5 ENTER.
(This just resets the list menus)

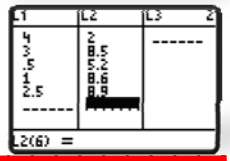
B) Next, press STAT ENTER

C) If there is OLD data already in the lists that needs to be cleared press the up arrow, ↑, to highlight L1 and then press CLEAR ENTER to clear out the old data. Do the same for L2 if it has OLD data that needs to be cleared.

To clear out **OLD** data, first highlight L1 and press CLEAR ENTER.

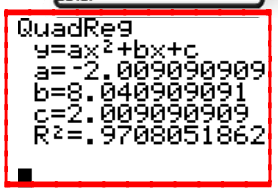






D) Next, enter the **Number of Employees** in **L1** and the **Gross Profit** in **L2**.

E) Return to the home screen by pressing 2nd MODE and then to calculate the quadratic regression press STAT ▸ 5 ENTER.



Fill in the blanks to complete the QUADRATIC REGRESSION equation.
(round to the nearest tenth)

$$y = \underset{a}{-2}\cdot x^2 + \underset{b}{8}\cdot x + \underset{c}{2}$$

You are trying to decide how many employees to hire. Let **x** represent the number of **employees** you hire; let **y** represent your profit in thousands of **dollars**. Use the equation you determined from the previous page.

$$y = \underset{a}{-2}\cdot x^2 + \underset{b}{8}\cdot x + \underset{c}{2}$$

5) Describe the shape and characteristic(s) of the graph of the function above based on the leading coefficient (a).

THE PARABOLA IS "UPSIDE DOWN" OR CONCAVE DOWN AND NARROW.

6) You need to graph this function in order to identify how many employees you need to hire that would maximize your profit. Describe how you would do this without using your calculator. MULTIPLE METHODS:

- A T-CHART COULD BE USED
- REWRITE THE PARABOLA IN VERTEX FORM BY COMPETING THE SQUARE
- FIND THE VERTEX USING THE LINE OF SYMMETRY $x = -b/2a$
- FIND THE ZEROS USING THE QUADRATIC FORMULA AND THEN THE VERTEX.

X	Y
0	2
1	8
2	10
3	8

$y = -2x^2 + 8x + 2$

$y = -2(x^2 - 4x + 4) + 8 + 2$

$y = -2(x-2)^2 + 10$

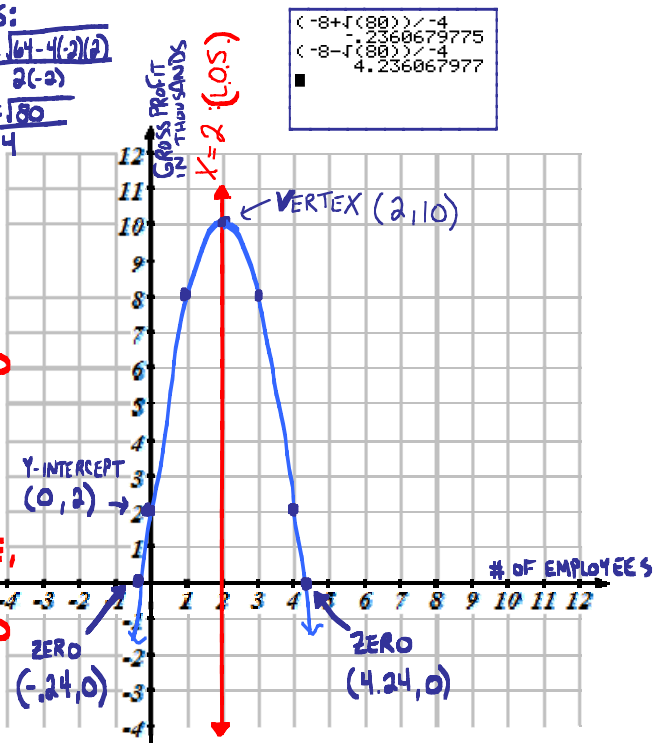
VERTEX: (2,10)

L.O.S. $x = \frac{-8}{2(-2)} = 2$

VERTEX: (2,10)

ZEROS: $y = \frac{-8 \pm \sqrt{64 - 4(-2)(2)}}{2(-2)}$

$= \frac{-8 \pm \sqrt{80}}{-4}$



7) Draw a sketch of the graph. **Label** the vertex, axis of symmetry, y-intercept, zeros, and axes (x-axis represents employees and y-axis represents thousands of dollars).

8) What does the vertex represent in this situation? What does the x-value represent? What does the y-value represent?

THE VERTEX REPRESENTS A MAXIMUM PROFIT OF $10,000 SHOULD BE EARNED BY HIRING 2 EMPLOYEES.

(2,10)

9) Based on your function, how much ***gross profit*** should be made with ***zero employees***? How might this be possible?

$y = -2(0)^2 + 8(0) + 2 = 2$ IF THE FUNCTION IS ACCURATE, IT WOULD SUGGEST $2000 IS MADE WITH 0 EMPLOYEES. IT COULD BE POSSIBLE TO HAVE 0 EMPLOYEES IF JUST THE OWNER WORKED

10) Describe what the zeros represent? Are these possible or do you think the domain should be restricted for this problem?

THE ZEROS, (-0.24, 0) (4.24, 0) WOULD REPRESENT THE NUMBER OF EMPLOYEES AT WHICH THE BUSINESS WOULD NOT MAKE ANY PROFIT. THE DOMAIN MIGHT HAVE TO BE RESTRICTED TO $x \geq 0$ OR UNDERSTAND NEGATIVE EMPLOYEES TO REPRESENT EMPLOYEES THAT WERE PAID BUT NEVER WORKED.

11) Make a table to show the relationship between employees and profit.

Number of Employees	Gross Profit
0	$2000
1	$8000
2	$10000
3	$8000
4	$2000
5	-$8000

12) What type of numbers most accurately describes the numbers you used for in the first column of your table? (Natural, Whole, Integers, Rational, Irrational, Real)

N: {1, 2, 3, ...} WHOLE: {0, 1, 2, ...} $\mathbb{Z}$ {..., -1, 0, 1, ...} $\mathbb{Q}$: IRRATIONALS: $\mathbb{R}$

13) Should the input of the function be Continuous? Explain.

AS LONG AS IT IS POSSIBLE TO HIRE PART-TIME EMPLOYEES FOR ANY LENGTH OF TIME IT COULD BE CONTINUOUS. IF INSTEAD THEY COULD ONLY BE HIRED AN HOUR AT A TIME THEN DISCRETE.

14) How many employees should you hire? Explain.

THE TREND SUGGESTS HIRING 2 EMPLOYEES BECAUSE THAT IS WHERE THE BUSINESS MAKES THE MOST MONEY.

TEACHER NOTE: IF TIME PERMITS, A GOOD CLASSROM DISCUSSION TOPIC TO INCLUDE IS THE DIFFERENCE BETWEEN THE CHART WITH ACTUAL DATA & PREDICTED DATA.

15) What can you say about the axis of symmetry?

IT IS THE LINE WHICH VERTICALLY DIVIDES THE PARABOLA IN HALF (SUCH THAT EITHER SIDE OF PARABOLA IS A REFLECTION OF THE OTHER). TO THE LEFT OF THE LINE OF SYMMETRY, THE FUNCTION REPRESENTS AN UNDERSTAFFED BUSINESS. TO THE RIGHT OF THE LINE OF SYMMETRY, THE FUNCTION REPRESENTS WHEN THE BUSINESS IS OVERSTAFFED.

16) If the owner is content with making $8000 in profit but would like to help her local economy by hiring as many employees as she can, how many employees should she hire?

THE TREND SUGGESTS THE OWNER SHOULD MAKE $8000 HIRING EITHER 1 OR 3 EMPLOYEES. SO, SHE SHOULD HIRE 3 EMPLOYEES.

17) Make a list of other quantitative variables you think a business owner should monitor for trends?

- COST OF ICECREAM
- PRICE CHARGED FOR ICE CREAM
- NUMBER OF ICECREAM PRODUCTS SOLD
- NUMBER OF CUSTOMER COMPLAINTS
- OVERHEAD COSTS
  - UTILITIES
  - INSURANCE
  - BUSINESS LICENSES/CERTIFICATES
- COST OF ADVERTISING
- EMPLOYEE PAY RATE
- HOURS OF OPERATION
- TAXES
- GROSS PROFIT
- NET PROFIT
- NUMBER OF EMPLOYEES

18) Provide two variables between which you think there should be a **linear** relationship. Explain your reasoning.

- THE COST OF THE ICE CREAM AND THE PRICE CHARGED FOR THE ICE CREAM COULD HAVE A LINEAR RELATIONSHIP. MANY BUSINESSES CHARGE A FLAT RATE ABOVE THEIR COST OR A STANDARD PERCENT INCREASE (EITHER WOULD CREATE A LINEAR RELATIONSHIP). USUALLY, AS THE COST INCREASES SO TO DOES THE PRICE CHARGED.
- NET PROFIT AND SALES TAX SHOULD ALSO BE LINEAR.

19) Provide two variables between which you think there should be a **quadratic** relationship (other than the number of employees and gross profit). Explain your reasoning.

- POSSIBLY A QUADRATIC RELATIONSHIP EXISTS BETWEEN THE PRICE CHARGED AND NUMBER OF ICE CREAMS SOLD. SOMETIMES, IF A PRODUCT IS TOO CHEAP NOT MANY PEOPLE WILL BUY IT BECAUSE THEY MIGHT THINK THE PRODUCT IS BAD. IF IT IS TOO EXPENSIVE, NOT TOO MANY WILL BUY IT BECAUSE THEY MIGHT THINK THEY ARE BEING TAKEN ADVANTAGE OF. IT IS ONLY WHEN THE PRICE IS WITH IN REASON THAT THE MOST WILL BE SOLD.
- HOURS OF OPERATION VS. GROSS PROFIT.

The Scoop on Selling Ice Cream - Interesting facts about the Ice Cream Business

- Each American consumes a yearly average of 23.2 quarts of ice cream, ice milk, sherbet, ices and other commercially produced frozen dairy products. How many gallons is this?
- The top three cites that purchase the most ice cream on a per capita basis are: Portland, Oregon; St. Louis, Missouri; and Seattle, Washington.
- Children of the ages two through 12, and adults age 45 plus, eat the most ice cream per person.
- 98 percent of all households purchase ice cream. (Source for the previous four statistics: MakeIceCream.com)