Henley Chocolates is famous for its mini chocolate truffles, which are packaged in foil covered boxes. The base of each box is created by cutting squares that are 4 centimeters on an edge from each corner of a rectangular piece of cardboard and folding the cardboard edges up to create a rectangular prism 4 centimeters deep. A matching lid is constructed in a similar manner, but, for this task, we focus on the base, which is illustrated in the diagrams below.
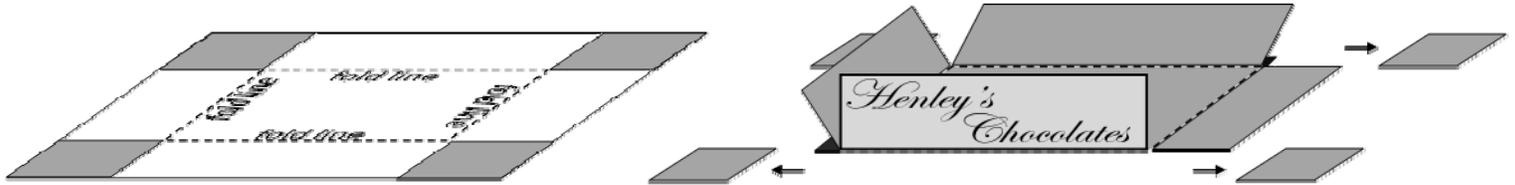
For the base of the truffle box, paper tape is used to join the cut edges at each corner. Then the inside and outside of the truffle box base are covered in foil.

Henley Chocolates sells to a variety retailers and creates specific box sizes in response to requests from particular clients. However, Henley Chocolates requires that their truffle boxes *always be __4 cm deep__* and that, in order to preserve the distinctive shape associated with Henley Chocolates, *the bottom of each truffle box be a rectangle that is __two and one-half times as long as it is wide__*.
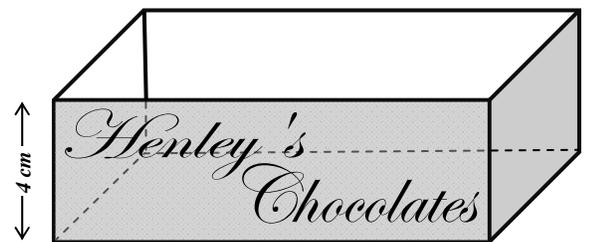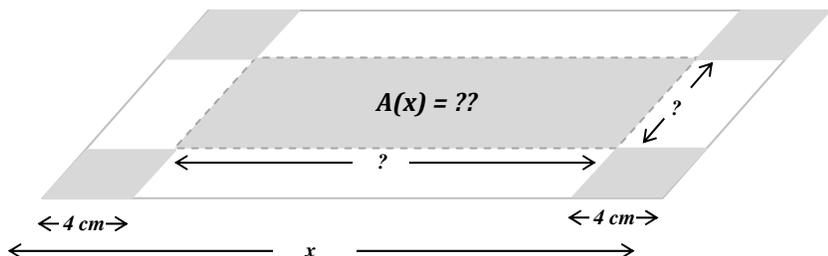
1. Henley Chocolates restricts box sizes to those which will hold plastic trays for a whole number of mini truffles. A box needs to be at least 2 centimeters wide to hold one row of mini truffles. Let *x* denote the length of a piece of cardboard from which a truffle box is made. What value of *x* corresponds to a finished box base for which the bottom is a rectangle that is 2 centimeters wide?
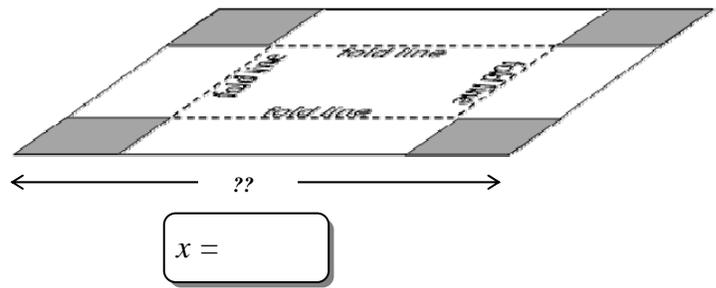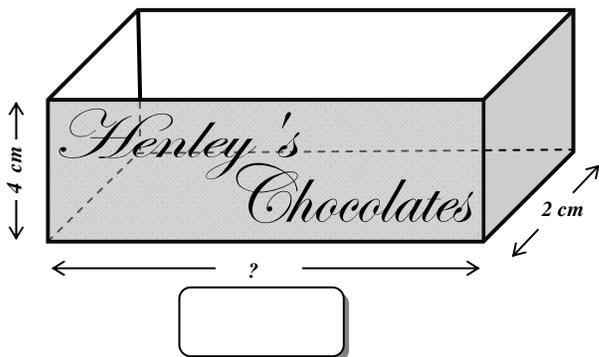
2. Henley Chocolates has a maximum size box of mini truffles that it will produce for retail sale. For this box, the bottom of the truffle box base is a rectangle that is 50 centimeters long. What are the dimensions of the piece of cardboard from which this size truffle box base is made?
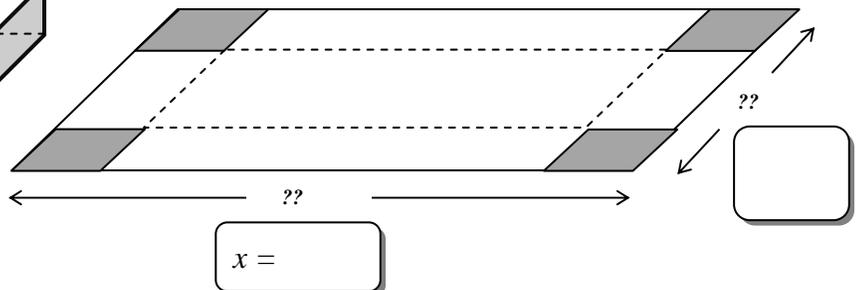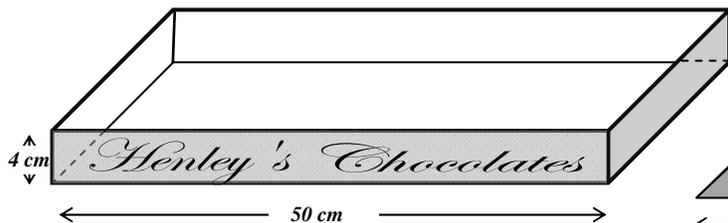
3. Since all of the mini truffle boxes are 4 centimeters deep, each box holds two layers of mini truffles. Thus, the number of truffles that can be packaged in a box depends on the number of truffles that can be in one layer, and, hence, on the area of the bottom of the box. Let $A(x)$ denote the area, in square centimeters, of the rectangular bottom of a truffle box base. Write a formula for $A(x)$ in terms of the length x, in centimeters, of the piece of cardboard from which the truffle box base is constructed.

A(x) = ??

?

←4 cm→ ←4 cm→

x

A(x) =

4. Although Henley Chocolates restricts truffle box sizes to those that fit the plastic trays for a whole number of mini truffles, the engineers responsible for box design find it simpler to study the function A on the domain of all real number values of x in the interval from the minimum value of x found in item 1 to the maximum value of x found in item 2. State this interval of x values as studied by the engineers at Henley Chocolates.

$$\square \le x \le \square$$

Is the domain continuous on this interval? Explain?

The next few items depart from Henley Chocolates to explore graph transformations that will give us insight about the function A for the area of the bottom of a mini truffle box. We will return to the function A in item 9.
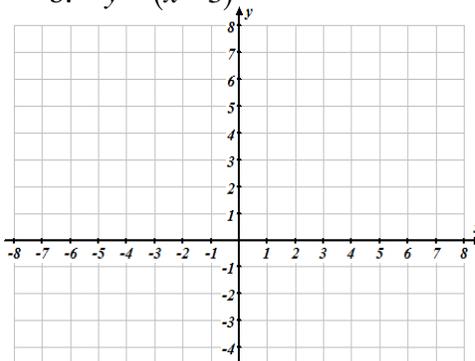
5. **Use technology to graph** each of the following functions on the same axes with the graph of the function f defined by $f(x) = x^2$. Use a new set of axes for each function listed below, but repeat the graph of f each time. For each function listed, sketch a graph and describe the rigid transformation of the graph of f that results in the graph of the given function. Make a conjecture about the graph of $y = (x-h)^2$, where h is any real number.
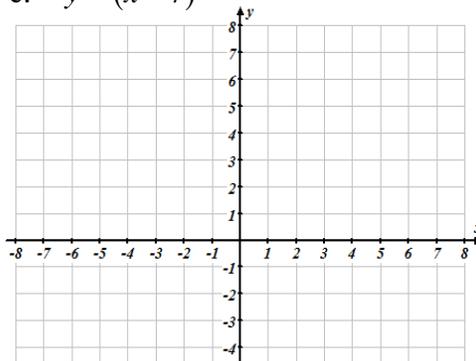
a. $y = x^2$
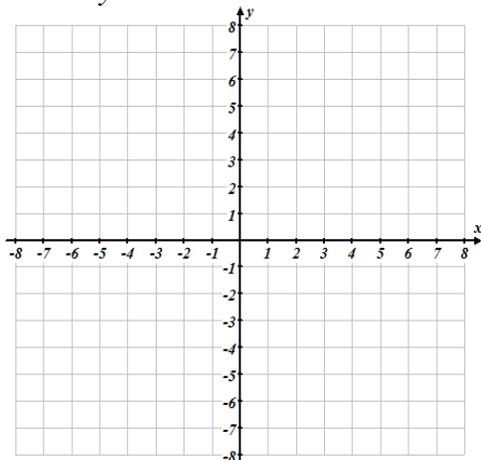
b. $y = (x-3)^2$

c. $y = (x-7)^2$
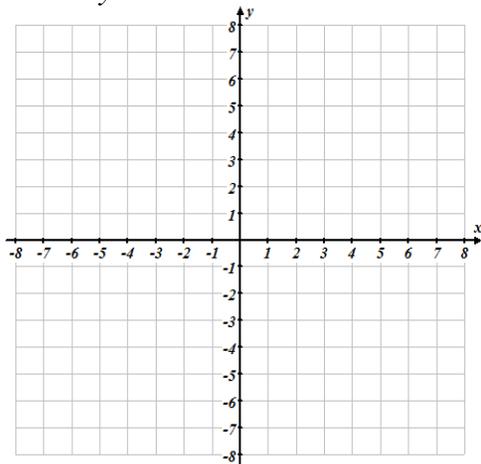
Description of transformation compared to $y = x^2$:
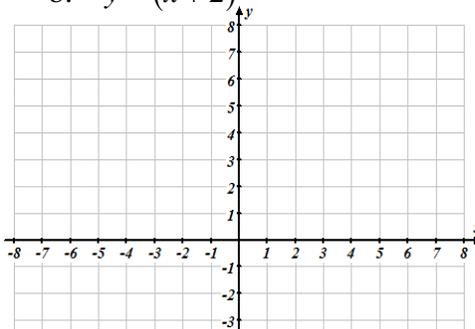
Description of transformation compared to $y = x^2$:

6. **Use technology to graph** each of the following functions on the same axes with the graph of the function f defined by $f(x) = x^2$. Use a new set of axes for each function listed below, but repeat the graph of f each time. For each function listed, describe a rigid transformation of the graph of f that results in the graph of the given function.
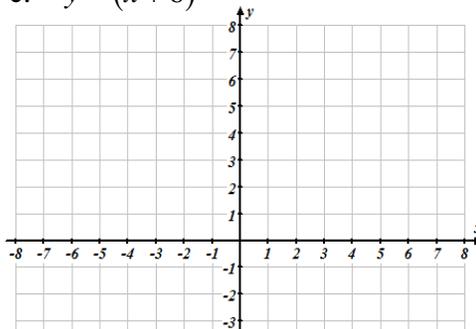
a. $y = x^2$

b. $y = (x+2)^2$

c. $y = (x+6)^2$

Description of transformation compared to $y = x^2$:

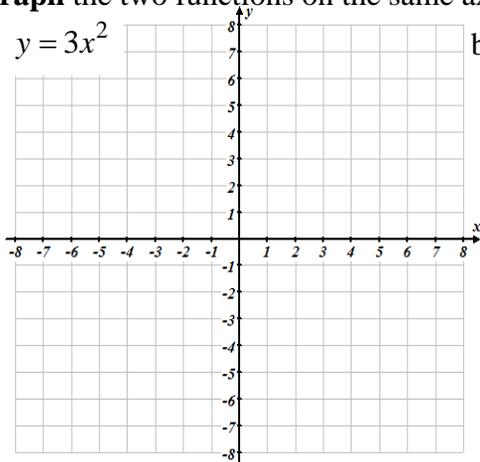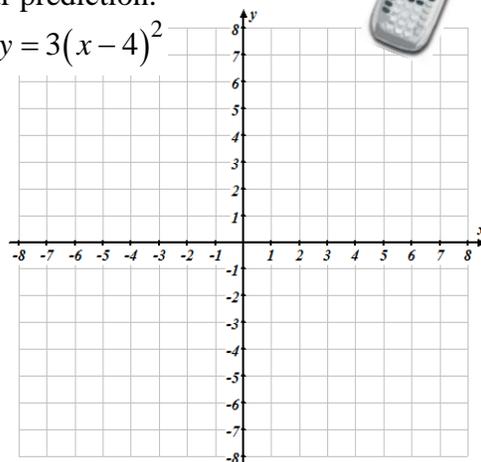Description of transformation compared to $y = x^2$:

7. For each pair of functions below, predict how you think the graphs will be related and then **use technology to graph** the two functions on the same axes and check your prediction.
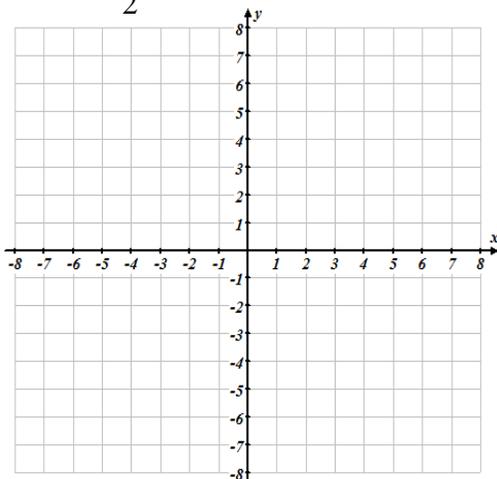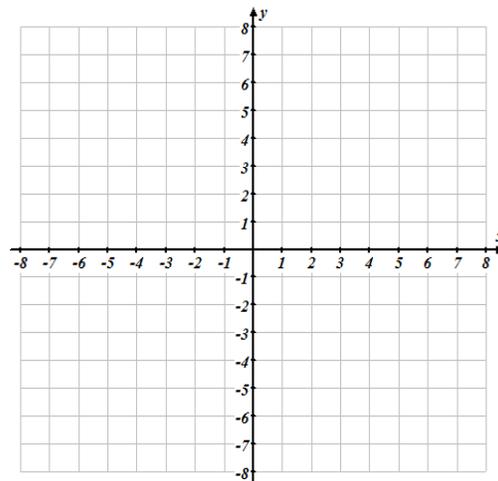
a. $y = x^2$ and $y = 3x^2$

b. $y = 3x^2$ and $y = 3(x-4)^2$

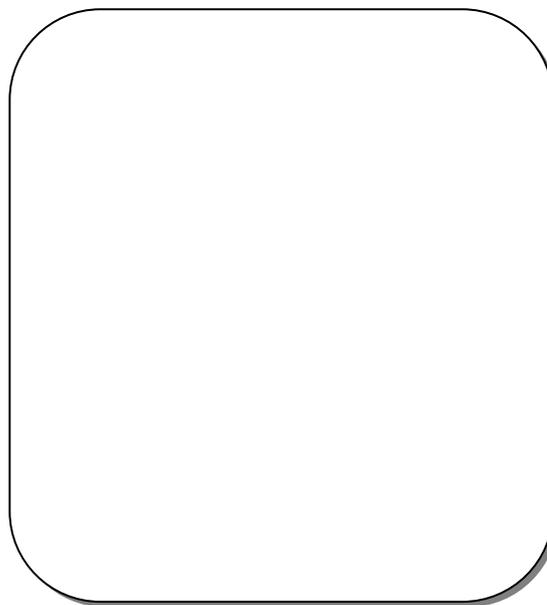c. $y = \dfrac{1}{2}x^2$ and $y = -\dfrac{1}{2}x^2 + 5$
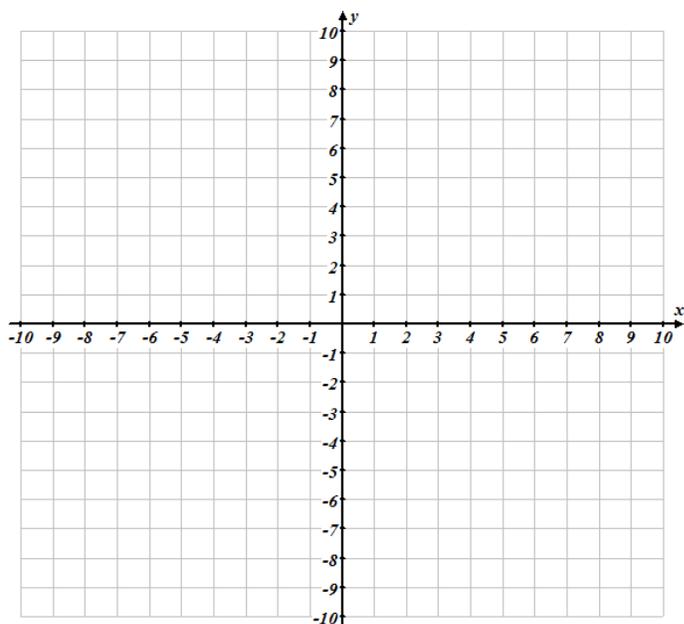
d. $y = 0.75x^2$ and $y = 2(x+3)^2 - 4$

Now we return to the function studied by the engineers at Henley Chocolates.

8. Consider again the function you determined in question #3, $A(x)$. Let $g(x)$ be the same as $A(x)$ except that the domain of g(x) is all real numbers. Sketch a graph of $f(x) = x^2$ and $g(x)$ on the graph below and describe the transformation from $f(x)$ to $g(x)$.

9. Using the graph above, describe the range of $g(x)$.

10. Remember that you found the domain of the function in item 4. What is the range of the function $A(x)$ and how does it differ from $g(x)$?

11. The engineers at Henley Chocolates responsible for box design have decided on two new box sizes that they will introduce for the next winter holiday season.
    a. The area of the bottom of the larger of the new boxes will be 640 square centimeters. Use the function $A(x)$ to write and solve an equation to find the length L of the cardboard need to make this new box.

    b. The area of the bottom of the smaller of the new boxes will be 40 square centimeters. Use the function A to write and solve an equation to find the length L of the cardboard need to make this new box.

12. How many mini-truffles do you think the engineers plan to put in each of the new boxes?