Section 5-4 Completing The Square & Quadratic Formula

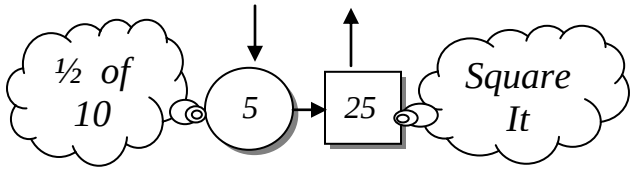
EASY

$x^2+10x-6=0$

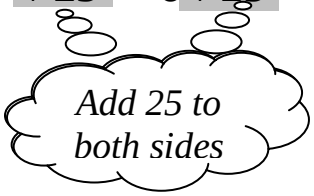
$x^2+10x-6=0$ *Move constant to the other side*

$+6 \quad +6$

$x^2+10x \quad =6$



$x^2+10x \quad +25 \quad =6+25$



$x^2+10x \quad +25 \quad =31$ *Easily Factors*

$(x+5)(x+5)=31$ *Can be re-written*

$(x+5)^2=31$ *Take the square root of both sides*

$\sqrt{(x+5)^2}=\sqrt{31}$ Don't forget $\pm$.

$x+5=\pm\sqrt{31}$ Isolate x

$-5 \quad -5$

$x=-5\pm\sqrt{31}$ *OR* $x\approx 0.5678$ *or* -10.5678

MEDIUM

$3x^2-15x+12=0$

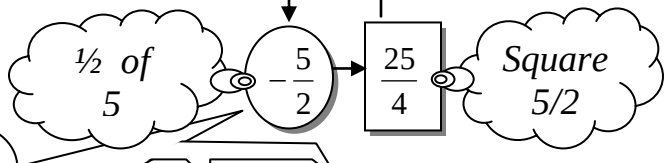
$3x^2-15x+9=0$ *Move constant to the other side*

$-9 \quad -9$

$3x^2-15x \quad =-9$ *Divide both sides by the leading coefficients*

$\frac{3x^2-15x}{3}=\frac{-9}{3}$

$x^2-5x \quad =-3 \longrightarrow x^2+10x \quad +\frac{25}{4}=-\frac{3}{1}+\frac{25}{4}$



Notice $-5/2$ it is the number that when added to itself is equal to –5 and when multiplied by itself is equal to $25/4$.

$\left(x-\frac{5}{2}\right)\left(x-\frac{5}{2}\right)=-\frac{12}{4}+\frac{25}{4} \longrightarrow \left(x-\frac{5}{2}\right)^2=\frac{13}{4}$

$\sqrt{\left(x-\frac{5}{2}\right)^2}=\pm\sqrt{\frac{13}{4}}$ $\quad x-\frac{5}{2}=\pm\frac{\sqrt{13}}{2}$ $\quad x=\frac{5}{2}\pm\frac{\sqrt{13}}{2}$ $\quad x=\frac{5\pm\sqrt{13}}{2}$

$x\approx 0.6972$ *or* -4.3028

Solve the following by completing the square.

1. $x^2 + 10x - 19 = 0$

2. $x^2 - 12x + 2 = 0$

3. $x^2 + 15x - 8 = 0$

4. $x^2 - 7x - 4 = 0$

5. $2x^2 - 8x - 14 = 0$

6. $3x^2 - 18x - 5 = 0$

Solve the following by completing the square.

7. $2x^2 - 10x - 3 = 0$

8. $3x^2 - 21x - 5 = 0$

9. $2x^2 - 5x - 8 = 0$

10. $3x^2 - 10x - 7 = 0$

Solve the following by completing the square. Derive the quadratic formula.

$$a \cdot x^2 + b \cdot x + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

Solve the following by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $x^2 + 10x - 19 = 0$

2. $x^2 - 12x + 2 = 0$

3. $x^2 + 15x - 8 = 0$

4. $x^2 - 7x - 4 = 0$

5. $2x^2 - 8x - 14 = 0$

6. $3x^2 - 18x - 5 = 0$

7. $2x^2 - 10x - 3 = 0$

8. $3x^2 - 21x - 5 = 0$

The DISCRIMINANT

The **discriminant** determines the type of solution **Rational** or **Irrational** and **Real** or **Imaginary**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow b^2 - 4ac = \textbf{\textit{discriminant}}$$

$b^2 - 4ac$

Positive Perfect Square Examples = 9, 25, 49	If the discriminate is a positive perfect square then the square root can be completely eliminated when the radical is simplified and no radical will be left over. So the solution can be written as **2** ***REAL*** and ***RATIONAL*** number.
Positive Non Square Examples = 7, 12, 22	If the discriminate is positive but NOT a perfect square then there will still be **2 REAL** solutions but the solutions will be **IRRATIONAL** since the radical cannot be completely eliminated.
Zero Examples = 0	If the discriminate is zero then there will be just **1 REAL RATIONAL** solution because adding or subtracting 0 is equivalent.
Negative Examples = – 3, – 9, – 12	If the discriminate is negative then there will be **2 IMAGINARY** solutions (there will be an ***i*** in the solution).

Describe the nature of the roots using the discriminant.

1. $2x^2 - 8x - 14 = 0$

2. $3x^2 + 2x + 8 = 0$

3. $3x^2 - 15x + 12 = 0$

4. $3x^2 - 10x - 7 = 0$

5. $8x^2 - 24x + 18 = 0$

6. $x^2 - 6x + 13 = 0$

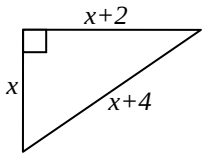
7. $12x^2 - 34x + 24 = 0$

8. Give a possible value for 'a' such that the solution of the quadratic would have 2 real rational solutions.

$$ax^2 - 5x - 2 = 0$$

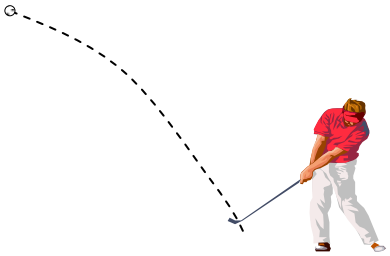
Applications

1. Find the value of x that would make the diagram below accurate.



2. A golf ball is hit with an initial vertical velocity of 80 fps

$$h = -16t^2 + 80t$$



a. How high is the ball after 2 seconds?

b. How many seconds would it take the ball to hit the ground (the height would be h=0)?

c. When will the ball reach 48 feet?

d. When does the ball reach a maximum height?

e. What is the average height from t = 1 to t = 2 seconds?

f. For what values of t is the domain appropriate?

3. A baseball his hit with an initial vertical velocity of 121 fps an the ball was struck 1 foot above ground.

$$h = -16t^2 + 121t + 1$$

a. How high is the ball after 2 seconds?

b. How many seconds would it take the ball to hit the ground (the height would be h=0)?

c. When will the ball reach 30 feet?

d. When does the ball reach a maximum height?

e. What is the average vertical velocity from t = 0.2 to t = 0.9 seconds?

4. A person at a framing store is making a frame mat to go around a picture. The mat is a uniform 2 inches around on each side. The picture's width is 5 less <u>than</u> twice the picture's height. The entire area of the frame with picture included is 221 square inches. What are the dimensions of the picture?