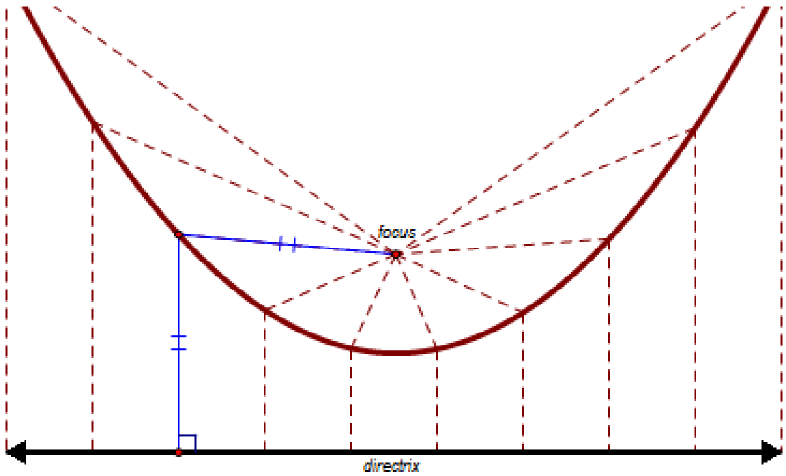
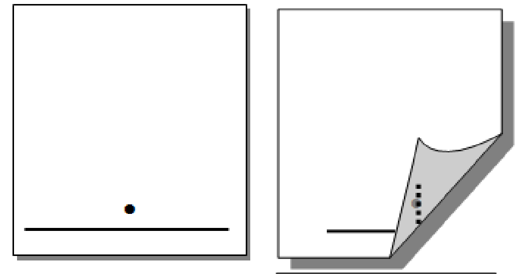
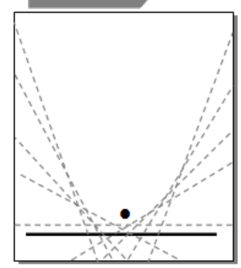
What is a parabola? It is geometrically defined by a set of points or locus of points that are equidistant from a point (the ***focus***) and a line (the ***directrix***).



Fold a Parabola



To see this idea visually try drawing a straight line at the bottom of a piece of paper with a ruler. Then, place a point in the middle just above the line as shown. Next, fold the paper multiple times in various locations so that the line folds on top of the point and make a crease.



The creases will outline a parabola. Why does this happen?

Parabolas can be found in many places in everyday life. A few examples are shown below. Can you guess where the focus point should be in the flash light or the satellite dish?

Fountains and Projectiles

Flash Light Reflectors

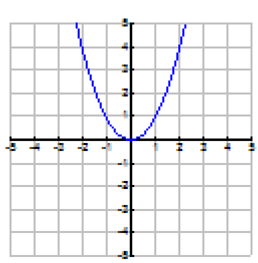
Algebraically, parabolas are usually defined in two different forms:

Standard Form and **Vertex Form**

$y = a \cdot x^2 + b \cdot x + c$ $\quad$ $y = a(x - h)^2 + k$

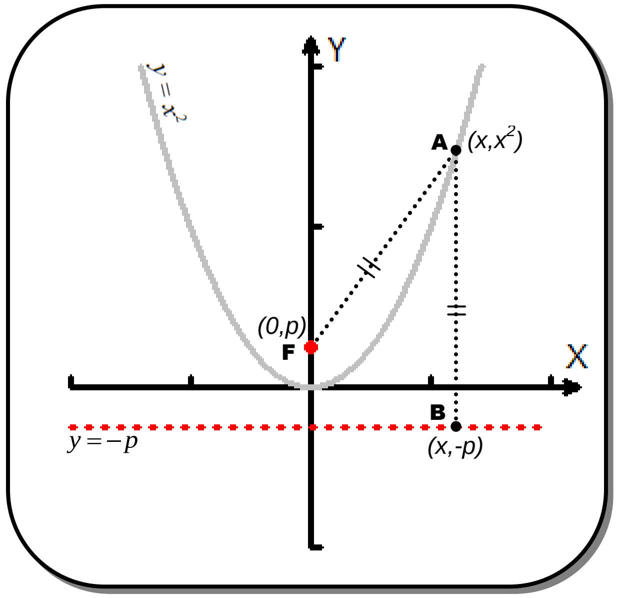
Lets start with the most basic graph of a parabola,

$y = x^2$



Satellite Dishes

So, where would the focus point and the directrix be in the basic equation of $y = x^2$. This is not a trivial task. We know that the directrix should be somewhere below the vertex and the distance from the vertex to the line should be the same as the distance from the vertex to the focus point.



Let's just put a arbitrary point on the y – axis above the vertex and call it the ***focus*** point at ***(0, p)***. We will need to figure out what p is to find the location of the focus. Then, we know that the directrix is the same distance away from the vertex but on the other side of the parabola and it is a line. So, it the ***directrix*** would need to be the line at $y = -p$.

The parabola is geometrically defined as the set of point equidistant from a point and a line. So, we already know algebraically the parabola is given by the equation $y = x^2$ which would suggest every point on the parabola is basically of the form (x, x^2) (eg. (1,1), (2, 4), (3,9), etc.). We described the point of the focus point as (0,p) and then, if you think about it carefully the point directly below the general point of the parabola shown in the diagram on the directrix would have to be the point (x,–p). Now, we can just use the distance formula to say that **AF = AB**.

$$\sqrt{(x_A - x_F)^2 + (y_A - y_F)^2} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

$$\sqrt{(x - 0)^2 + (x^2 - p)^2} = \sqrt{(x - x)^2 + (x^2 - {}^-p)^2}$$

We can square both sides:

$$(x - 0)^2 + (x^2 - p)^2 = (x - x)^2 + (x^2 - {}^-p)^2$$

Clean things up a bit:

$$x^2 + (x^2 - p)^2 = 0^2 + (x^2 + p)^2$$

Expand (use F.O.I.L. if necessary):

$$x^2 + x^4 - 2x^2p + p^2 = x^4 + 2x^2p + p^2$$

Cross out items on both sides (subtract from both sides)

$$x^2 + \cancel{x^4} - 2x^2p + \cancel{p^2} = \cancel{x^4} + 2x^2p + \cancel{p^2}$$

Move the $2x^2p$ to the right:

$$x^2 - 2x^2p = 2x^2p$$
$$\underline{\quad +2x^2p \quad +2x^2p}$$
$$x^2 \quad = \quad 4x^2p$$

Solve for p

$$\frac{x^2}{4x^2} = \frac{4x^2p}{4x^2}$$

Which reduces

$$\frac{1}{4} = p$$

So the **focus** is located at $\left(0, \frac{1}{4}\right)$ and the **directrix** is located at $y = -\frac{1}{4}$

It turns out doing nearly the same thing for the general vertex form of the parabola,

$$y = a(x-h)^2 + k$$

we can show in a similar manner that the focus is located 'inside the mouth' of the parabola a vertical distance of $\frac{1}{4a}$ from the vertex. Where as the directrix is located the same distance away from the vertex on the other side of the parabola from the focus.

Example #1: Find the vertex form, the focus, and the directrix of each of the following parabola.

A. $y = x^2 + 6x + 2$

**First complete the square

$$y = x^2 + 6x \qquad + 2$$

3 (½ it) → 9 (Sqr it)

**Balance

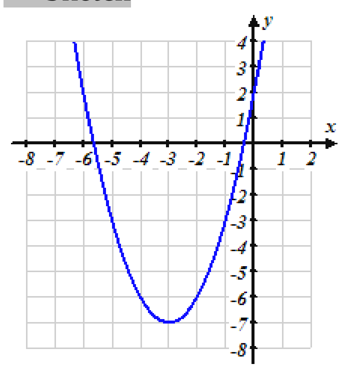
$$y = \underline{x^2 + 6x \underline{+9}} \; \underline{\underline{-9} + 2}$$

**Factor and simplify

$$y = (x+3)^2 - 7$$

***** VERTEX at (–3, –7)***

** Sketch



FOCUS is located at :

$$\left(\boxed{-3}, \boxed{-7 + \frac{1}{4 \cdot 1}} \right) = (-3, -6.75)$$

Came from the vertex. Came from the 1/(4a).

DIRECTRIX is located at :

$$y = -7 - \frac{1}{4 \cdot 1}$$

$$y = -7.25$$

B. $y = -2x^2 + 10x + 6$

**First complete the square

$$y = -2\left(x^2 - 5x \qquad\right) \qquad + 6$$

-5/2 (½ it) → 25/4 (Sqr it)

**Balance

$$y = -2\left(x^2 - 5x + \frac{25}{4} \right) + \frac{50}{4} + 6$$

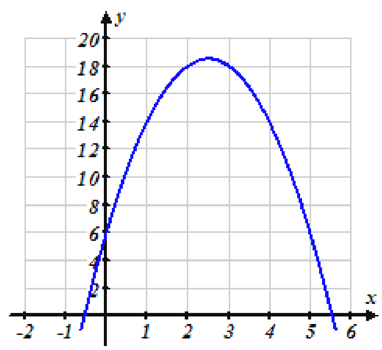
This is (+50/4) because everything in the parenthesis is multiplied by (-2)

**Factor and simplify

$$y = -2\left(x - \frac{5}{2} \right)^2 + \frac{37}{2}$$

***** VERTEX at (2.5, 18.5)***

**Sketch



FOCUS is located at :

$$\left(\boxed{2.5}, \boxed{18.5 + \frac{1}{4 \cdot -2}} \right) = (2.5, 18.375)$$

Came from the vertex. Came from the 1/(4a).

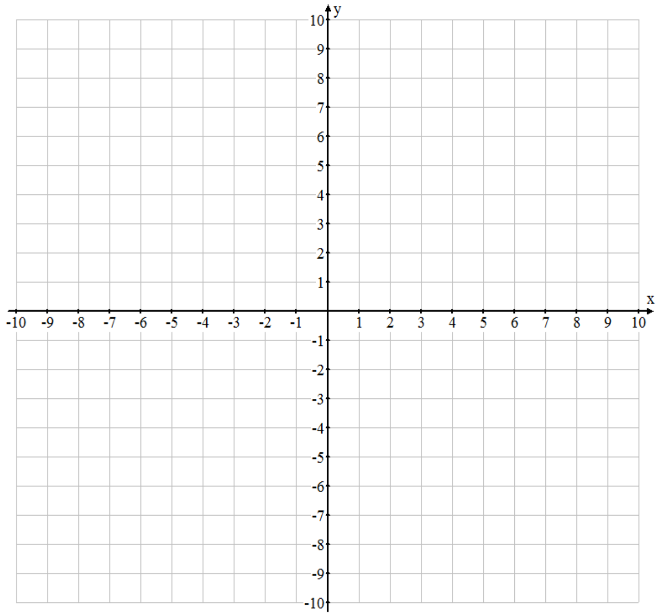
DIRECTRIX is located at :

$$y = 18.5 - \frac{1}{4 \cdot -2}$$

$$y = 18.625$$

4. ***Parabola*** (forward). Given $y = x^2 + 10x + 18$

A. Find the equation of the parabola in **vertex** form:



B. Location of **Vertex**:

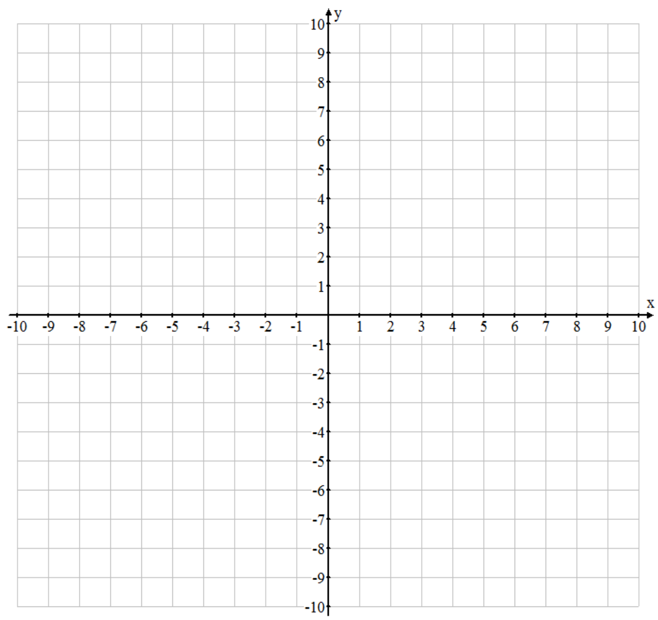
C. Location of **Focus**:

D. Location of **Directrix**:

E. Sketch a **Graph**.

5. ***Parabola*** (forward). Given $y = 3x^2 - 18x + 24$

A. Find the equation of the parabola in **vertex** form:



B. Location of **Vertex**:

C. Location of **Focus**:

E. Location of **Directrix**:

E. Sketch a **Graph**.

6. ***Parabola*** (forward). Given $y = -2x^2 - 8x - 1$

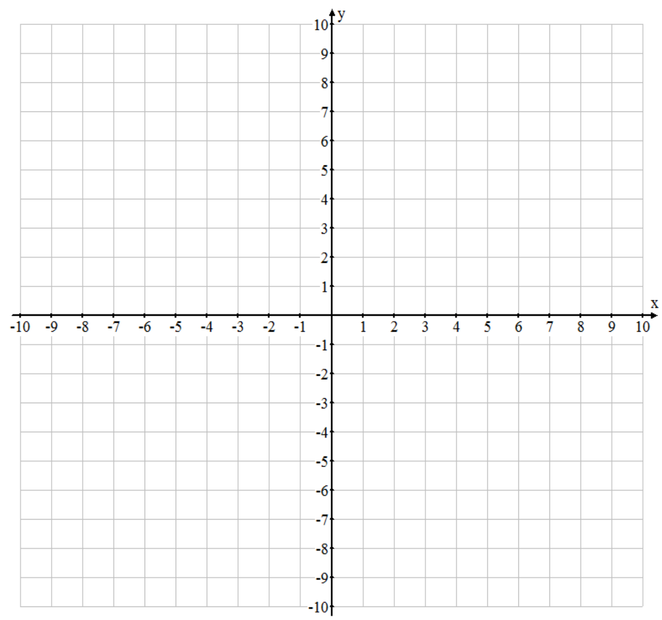
A. Find the equation of the parabola in **vertex** form:

B. Location of **Vertex**:

C. Location of **Focus**:

F. Location of **Directrix**:

E. Sketch a **Graph**.



7. ***Parabola*** (forward). Given $y = \frac{1}{4}x^2 - 3x + 2$

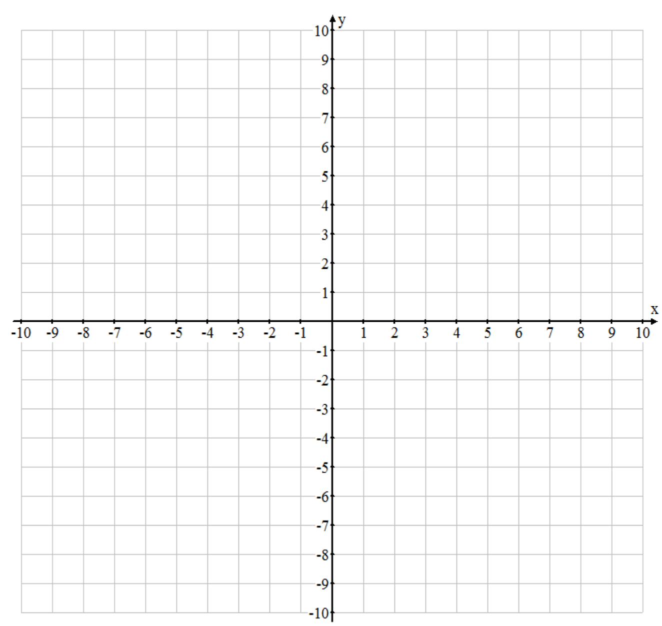
A. Find the equation of the parabola in **vertex** form:

B. Location of **Vertex**:

C. Location of **Focus**:

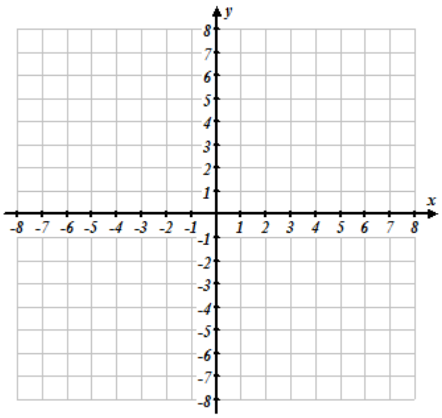
G. Location of **Directrix**:

E. Sketch a **Graph**.

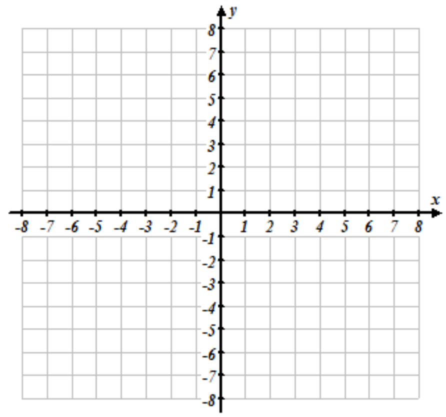


8. ***Horizontal Parabolas*** Graph the following:

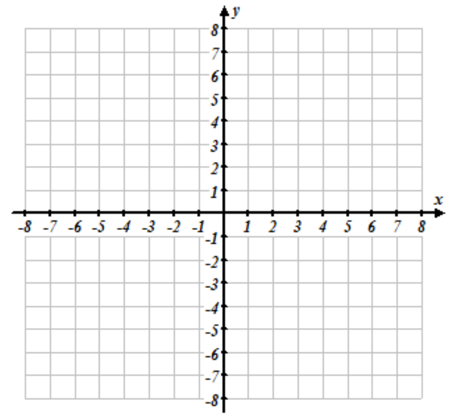
A. $x+1=(y+2)^2$



B. $x-4=-2(y-3)^2$



C. $x+3=\frac{1}{2}(y+2)^2$



9. ***Parabola*** (forward). Given $x=2y^2-12y+20$

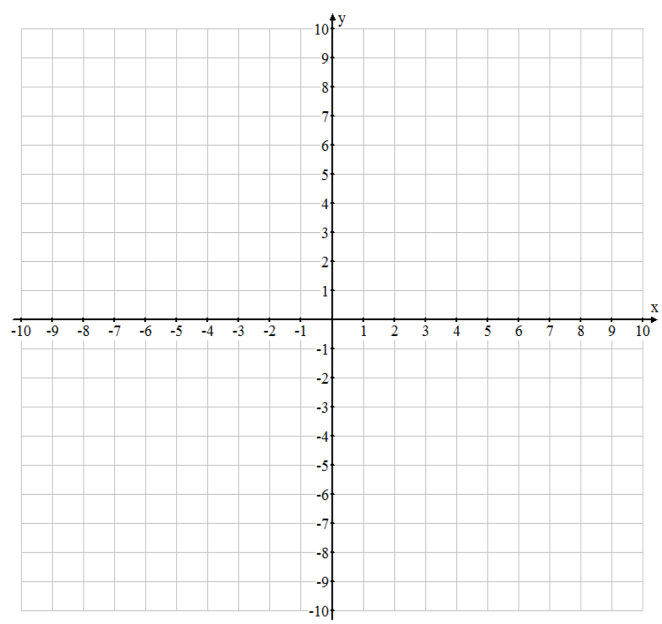
A. Find the equation of the parabola in **vertex** form:

B. Location of **Vertex**:

C. Location of **Focus**:

D. Location of **Directrix**:

E. Sketch a **Graph**.



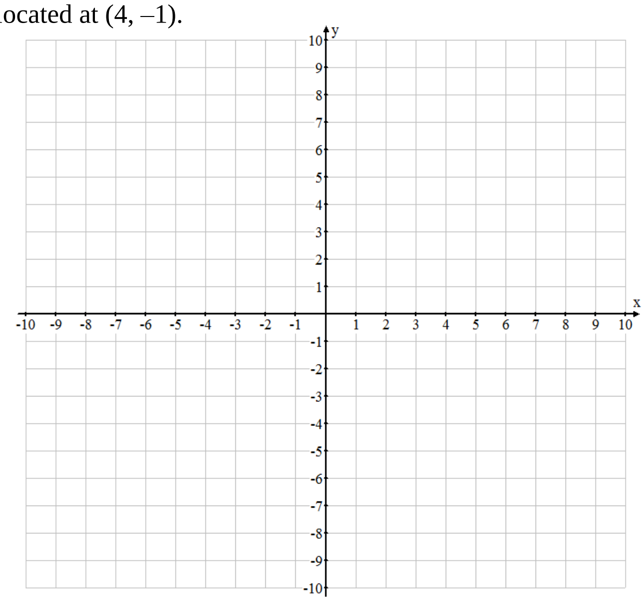
10. ***Parabola*** (backwards)

A vertex of a parabola is located at (4, –4) and the focus is located at (4, –1).

A. Find the **Direcrix**:

B. Find the **equation** of the parabola in vertex form:

C. Sketch a **Graph** of the Parabola



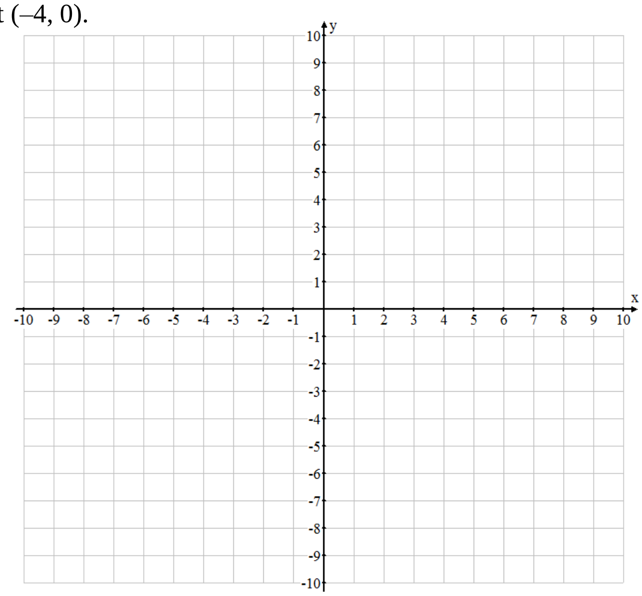
11. ***Parabola*** (backwards)

A directrix of a parabola is y = –2 and the focus is located at (–4, 0).

A. Find the **Vertex Point**:

B. Find the **equation** of the parabola in vertex form:

C. Sketch a **Graph** of the Parabola



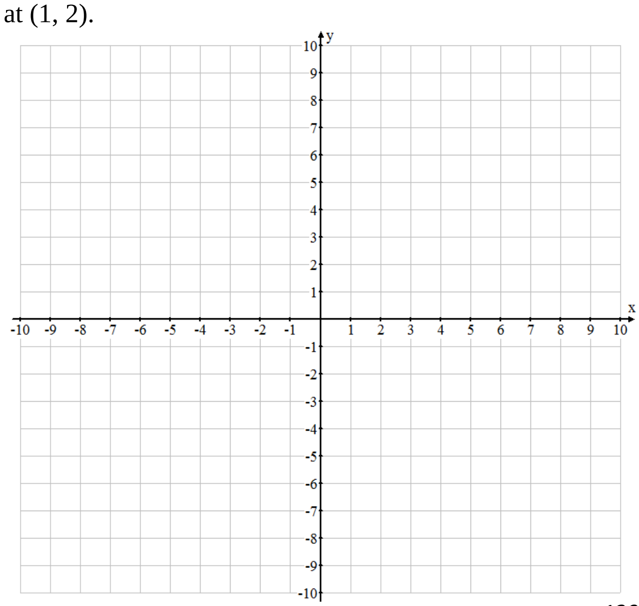
12. ***Parabola*** (backwards)

A directrix of a parabola is y = 1.5 and the vertex is located at (1, 2).

A. Find the **Focus**:

B. Find the **equation** of the parabola in vertex form:

C. Sketch a **Graph** of the Parabola



13. A parabola has a focus at (– 3, 6) and a directrix of y = 4 . For which value of *a* does the point (*a*, 6) lie on the parabola?

14. A parabola has a vertex at (0, – 3) and a directrix of y = – 2.75 . For which value of *b* does the point (– *2* , *b*) lie on the parabola?

15. A parabola has a vertex at (2, 4) and a focus at (2, 3.5). For which value of *c* does the point (*4* , *c*) lie on the parabola?