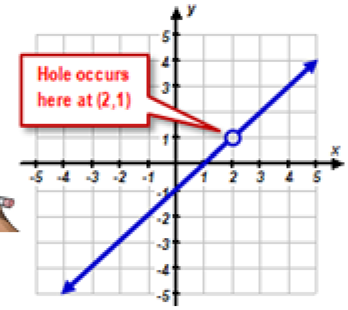
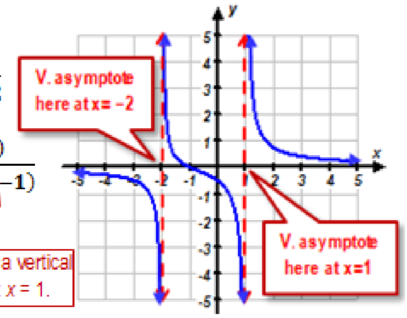
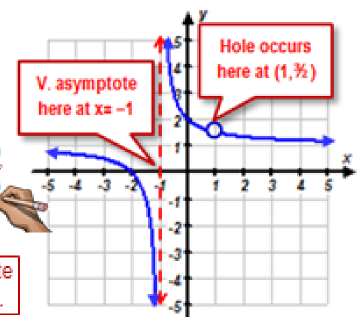
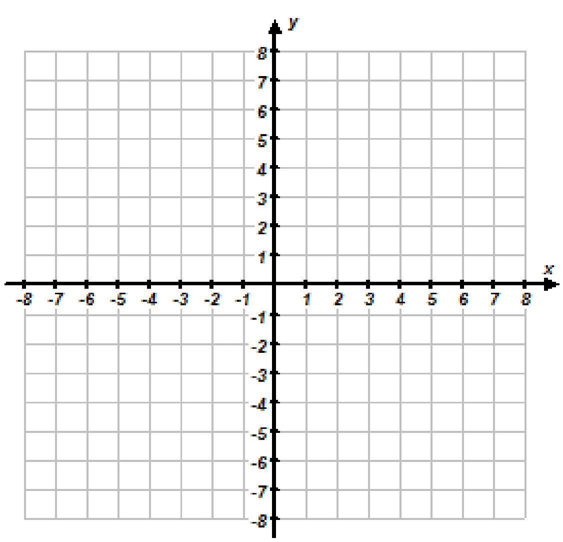
Sec 4.7 – Rational & Radical Relationships
Graphing Rational Functions

Name:

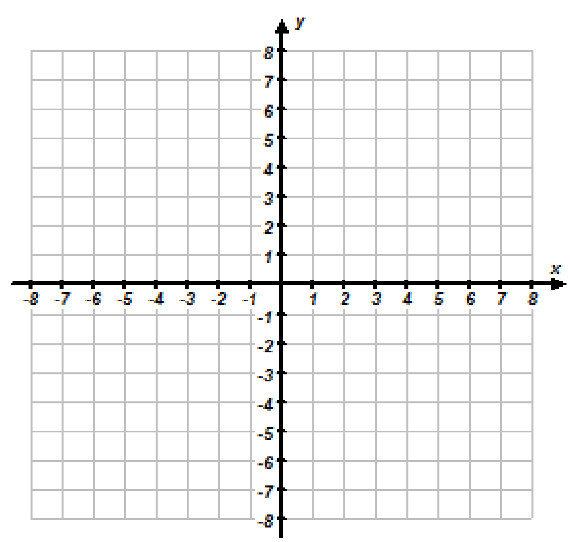
Characteristic	Description	Example
Hole *(Point Discontinuity)*	A hole **usually** occurs in the graph of a rational function when a linear factor in the numerator and denominator "divide out". The result is the same as the graph of the simplified function but with a missing point in the graph.	**Graph:** $f(x)=\frac{x^2-3x+2}{x-2}$ **Simplifies:** $f(x)=\frac{(x-2)(x-1)}{(x-2)}$ This will create a hole at $x = 2$. 
Vertical Asymptote *(Infinite Discontinuity)*	A vertical asymptote occurs any time a linear factor of the denominator doesn't "divide out" with a factor in the numerator.	**Graph:** $f(x)=\frac{x+1}{x^2+x-2}$ **Simplifies:** $f(x)=\frac{(x+1)}{(x+2)(x-1)}$ This will create a vertical asymptote at $x = -2$. This will create a vertical asymptote at $x = 1$. 
Vertical Asymptote & Hole	To have both a hole and a vertical asymptote the rational function must have at least one linear factor that divides out and one linear factor that does not.	**Graph:** $f(x)=\frac{x^2+x-2}{x^2-1}$ **Simplifies:** $f(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ This will create a vertical asymptote at $x = -1$. This will create hole at $x = 1$. 

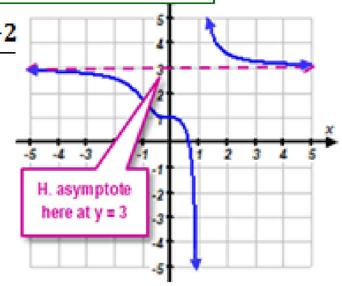
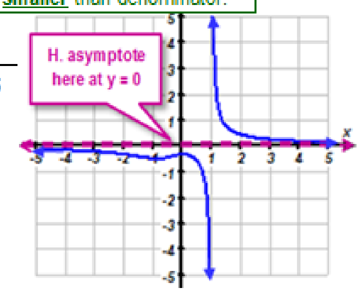
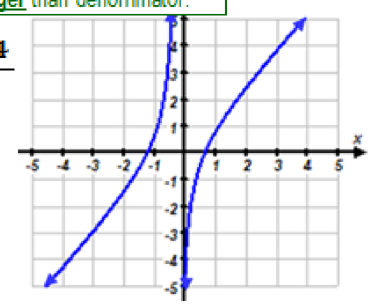
Sketch a graph of the following rational functions. Label any holes or vertical asymptotes. Use your calculator for additional assistance.

1. $f(x)=\frac{x^2+2x-3}{x-1}$



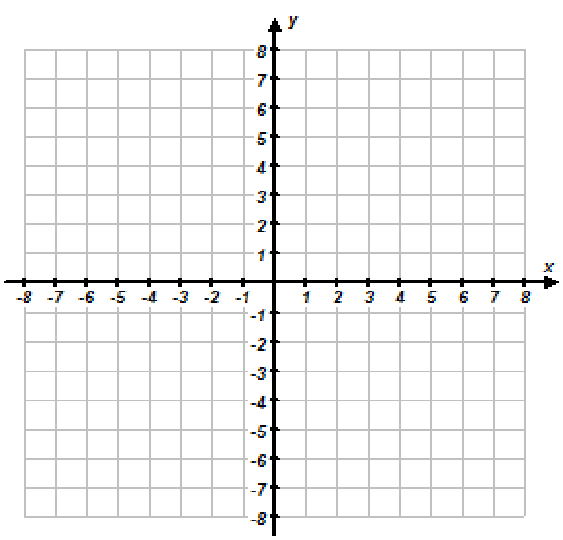
2. $f(x)=\frac{x^2+3x-4}{x^2+2x-8}$



Potential Horizontal Asymptotes	Description	Example
Case #1:	A rational function that has a numerator polynomial with the **<u>same degree</u>** as the polynomial in the denominator creates a horizontal asymptote that passes through the y-axis at the quotient of the leading coefficients.	The degree of the numerator & denominator are the **<u>same</u>** **Graph:** $f(x) = \frac{6x^3 + x^2 - 2}{2x^3 - 2}$ **Analyze:** $f(x) = \frac{6x^3 + x^2 - 2}{2x^3 - 2}$ **Asymptote:** $y = \frac{6}{2}$ **or** $y = 3$ 
Case #2:	A rational function that has a polynomial in the **<u>numerator that has a smaller degree</u>** than the degree of the polynomial in the denominator creates a horizontal asymptote at y =0.	The degree of the **<u>numerator is smaller</u>** than denominator. **Graph:** $f(x) = \frac{2x^2 + 1}{3x^3 - 3}$ **Asymptote:** $y = 0$ 
Case #3:	A rational function that has a polynomial in the **<u>numerator that has a larger degree</u>** than the degree of the polynomial in the denominator does not have a horizontal asymptote.	The degree of the **<u>numerator is larger</u>** than denominator. **Graph:** $f(x) = \frac{5x^2 + 3x - 4}{4x^1 + 1}$ **Asymptote:** **No Horizontal Asymptote** 

Sketch a graph of the following rational functions. Label any vertical asymptotes, horizontal asymptotes, or holes. Use your calculator for additional assistance.

3. $f(x) = \frac{x-4}{x^2-2x-8}$



4. $f(x) = \frac{2x^2-8}{x^2+x-6}$