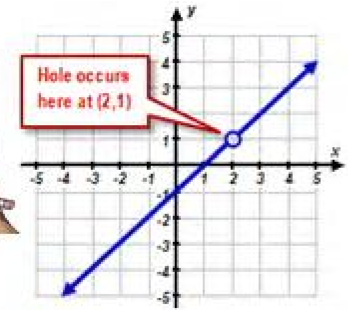
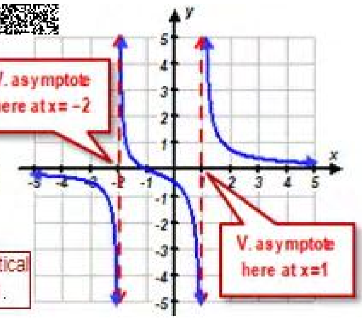
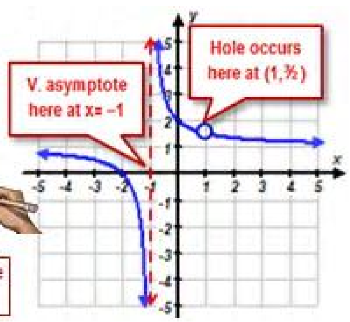
Characteristic	Description	Example
Hole *(Point Discontinuity)*	A hole **usually** occurs in the graph of a rational function when a linear factor in the numerator and denominator “divide out”. The result is the same as the graph of the simplified function but with a missing point in the graph.	**Graph:** $f(x) = \frac{x^2-3x+2}{x-2}$ **Simplifies:** $f(x) = \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}}$ This will create a hole at $x = 2$. $X-2 \neq 0$ $+2 \quad +2$ $X \neq 2$  Hole occurs here at (2,1)
Vertical Asymptote *(Infinite Discontinuity)*	A vertical asymptote occurs any time a linear factor of the denominator doesn’t “divide out” with a factor in the numerator. $X-1 \neq 0$ $+1 \quad +1$ $X \neq 1$ $X+2 \neq 0$ $-2 \quad -2$ $X \neq -2$	**Graph:** $f(x) = \frac{x+1}{x^2+x-2}$ **Simplifies:** $f(x) = \frac{(x+1)}{(x+2)(x-1)}$ This will create a vertical asymptote at $x = -2$. This will create a vertical asymptote at $x = 1$.  V. asymptote here at x = -2 V. asymptote here at x=1
Vertical Asymptote & Hole	To have both a hole and a vertical asymptote the rational function must have at least one linear factor that divides out and one linear factor that does not.	**Graph:** $f(x) = \frac{x^2+x-2}{x^2-1}$ **Simplifies:** $f(x) = \frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$ This will create a vertical asymptote at $x = -1$. This will create hole at $x = 1$.  V. asymptote here at x= -1 Hole occurs here at (1, ½)

Sketch a graph of the following rational functions. Label any holes or vertical asymptotes. Use your calculator for additional assistance.

1. $f(x) = \frac{x^2+2x-3}{x-1} = \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}}$

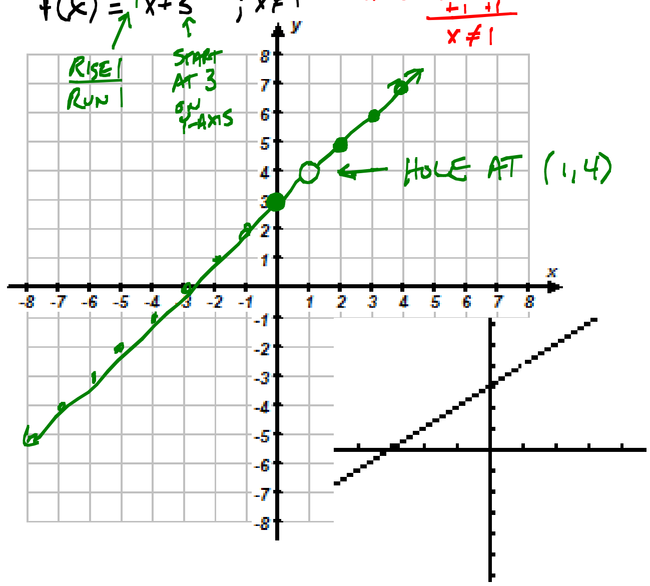
-3: $1,-3$; $-1,3$

HOLE $X-1 \neq 0$, $+1 \; +1$, $X \neq 1$

$f(x) = 1X+3 \; ; X \neq 1$

RISE 1 RUN 1

START AT 3 ON Y-AXIS



HOLE AT (1,4)

2. $f(x) = \frac{x^2+3x-4}{x^2+2x-8} = \frac{(x-1)\cancel{(x+4)}}{(x-2)\cancel{(x+4)}} = \frac{x-1}{x-2}$

-4: $1,-4$; $-1,4$; $2,-2$

-8: $1,-8$; $-1,8$; $2,-4$; $-2,4$

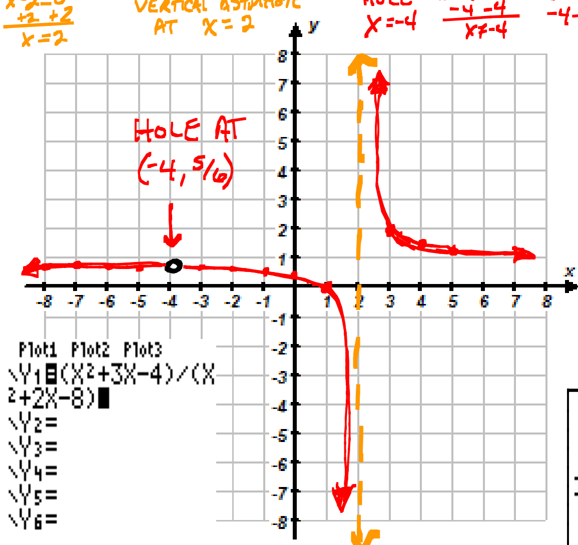
$X-2=0$, $+2 \; +2$, $X=2$

VERTICAL ASYMPTOTE AT $X=2$

HOLE $X=-4$ $X+4 \neq 0$, $-4 \; -4$, $X \neq -4$

$\frac{-4-1}{-4-2} = \frac{-5}{-6}$

HOLE AT $(-4, 5/6)$

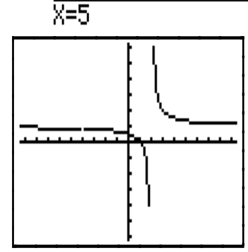


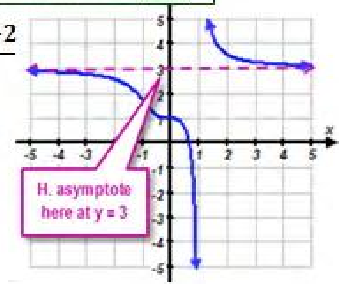
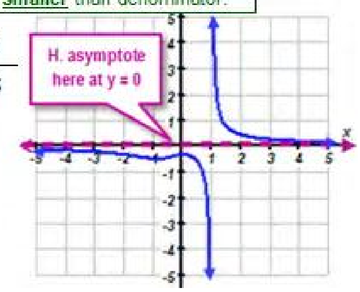
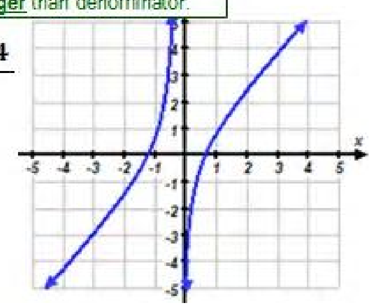
```
Plot1 Plot2 Plot3
\Y1=(X²+3X-4)/(X²+2X-8)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
-8	.9
-7	.88889
-6	.875
-5	.85714
-4	ERROR
-3	.8
-2	.75

X	Y1
-1	.66667
0	.5
1	0
2	ERROR
3	2
4	1.5
5	1.3333

X=5



Potential Horizontal Asymptotes	Description	Example
Case #1:	A rational function that has a numerator polynomial with the **<u>same degree</u>** as the polynomial in the denominator creates a horizontal asymptote that passes through the y-axis at the quotient of the leading coefficients.	The degree of the numerator & denominator are the <u>same</u> **Graph:** $f(x)=\dfrac{6x^3+x^2-2}{2x^3-2}$ **Analyze:** $f(x)=\dfrac{6x^3+x^2-2}{2x^3-2}$ **Asymptote:** $y=\frac{6}{2}$ or $y=3$ 
Case #2:	A rational function that has a polynomial in the **<u>numerator that has a smaller degree</u>** than the degree of the polynomial in the denominator creates a horizontal asymptote at y =0.	The degree of the <u>numerator is smaller</u> than denominator. **Graph:** $f(x)=\dfrac{2x^2+1}{3x^3-3}$ **Asymptote:** $y=0$ 
Case #3:	A rational function that has a polynomial in the **<u>numerator that has a larger degree</u>** than the degree of the polynomial in the denominator does not have a horizontal asymptote.	The degree of the <u>numerator is larger</u> than denominator. **Graph:** $f(x)=\dfrac{5x^2+3x-4}{4x^1+1}$ **Asymptote:** No Horizontal Asymptote 

Sketch a graph of the following rational functions. Label any vertical asymptotes, horizontal asymptotes, or holes. Use your calculator for additional assistance.

3. $f(x)=\dfrac{x-4}{x^2-2x-8} = \dfrac{\cancel{(x-4)}}{(x+2)\cancel{(x-4)}} = \dfrac{1}{x+2}$

-8
1, -8
-1, 8
2, -4
-2, 4

$x-4=0$
$+4\ \ +4$
$x=4$

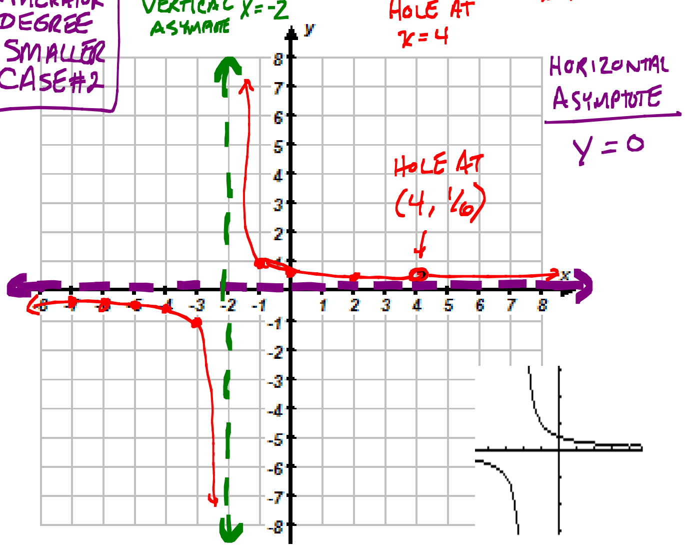
NUMERATOR DEGREE SMALLER CASE#2

VERTICAL ASYMPTOTE $x=-2$

HOLE AT $x=4$

HORIZONTAL ASYMPTOTE
$y=0$

HOLE AT $(4, \frac{1}{6})$



4. $f(x)=\dfrac{2x^2-8}{x^2+x-6} = \dfrac{2(x^2-4)}{(x-2)(x+3)} = \dfrac{2(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x+3)} = \dfrac{2(x+2)}{(x+3)}$

-6
1, -6
-1, 6
2, -3
-2, 3

SAME DEGREE CASE #1

HOLE AT $x=2$

V. ASYM. AT $x=-3$

HOLE AT
(2, 1.6)

HORIZONTAL ASYMPTOTE
$y=\frac{2}{1}$; $y=2$

HOLE
$\dfrac{2(2+2)}{2+3}=\dfrac{8}{5}$
$=1.6$