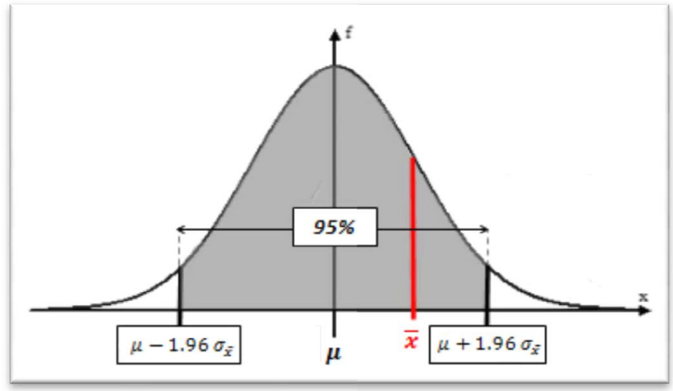
Name:

Using the Central Limit Theorem we can construct interval estimates of the population mean with a certain level of confidence. Let's construct a 95% confidence interval.



First notice that there is an approximate area of 0.95 between z = – 1.96 and z = 1.96.

We also know that for relatively large sample sizes that the sample means are normally distributed with a mean of $\mu_{\bar{x}} = \mu$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (also known as the standard error).

So, there is a 95% chance that sample will have mean somewhere between $(\mu - 1.96\,\sigma_{\bar{x}})$ and $(\mu + 1.96\,\sigma_{\bar{x}})$ which we can write more algebraically as:

Population Mean | Standard Error | Sample Mean | Population Mean | Standard Error

$$\cancel{\mu} - 1.96\,\sigma_{\bar{x}} < \bar{x} < \cancel{\mu} + 1.96\,\sigma_{\bar{x}}$$
$$-\cancel{\mu} \qquad -\mu \qquad -\cancel{\mu}$$

$$-1.96\sigma_{\bar{x}} < \cancel{\bar{x}} - \mu < 1.96\,\sigma_{\bar{x}}$$
$$-\bar{x} \qquad -\cancel{\bar{x}} \qquad -\bar{x}$$

$$-\bar{x} - 1.96\sigma_{\bar{x}} < -\mu < -\bar{x} + 1.96\sigma_{\bar{x}}$$
$$-1 \qquad -1 \qquad -1$$

Then, with some algebra that can be rewritten as:

$$\bar{x} + 1.96\sigma_{\bar{x}} > \mu > \bar{x} - 1.96\sigma_{\bar{x}}$$

$$\bar{x} - 1.96\,\sigma_{\bar{x}} < \mu < \bar{x} + 1.96\,\sigma_{\bar{x}}$$

This suggests we can say with 95% probability or confidence that the true mean is somewhere between the two calculated boundaries.

1. Try constructing a 95% confidence interval for the true mean price of a gallon of gas given the following information. A lawn company looked at a sample of 30 days out of the previous year and found the mean cost of a gallon of gas was \$2.70. With a 95% confidence what is the mean price of a gallon of gas for the year? You may assume the population standard deviation is \$0.45.

   a. Determine the Standard Error, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.45}{\sqrt{30}} \approx .0822$

   b. Determine the Margin of Error, $E = (Critical\ Value) \cdot \sigma_{\bar{x}}$
   (The Critical Value for 95% Confidence is 1.96)
   $(1.96)\ (.0822) \approx 0.16$

   c. Construct the confidence interval. Write the interval 3 different ways (Using ±, interval notation, and set notation)

   USING ±: TRUE POPULATION MEAN IS \$2.70 WITH AN ERROR OF ± \$0.16.

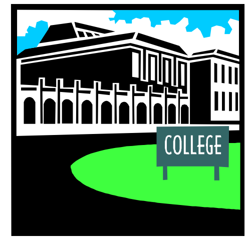
   USING INTERVAL: WITH 95% CONFIDENCE THE POPULATION MEAN IS IN THE INTERVAL (2.54, 2.86)

   USING SET NOTATION: $\$2.54 < \mu < \$2.86$ WITH 95% CONFIDENCE

2. It was determined with 95% confidence that 44% would vote for the republican candidate for president in city with an error of ±4%. If 2400 voters came to vote in that city, determine the possible range of voters that voted for the republican candidate. 44% - 4% = 40% | (0.40) (2400) = 960 | BETWEEN 960 AND 1152

44% + 4% = 48% | (0.48) (2400) = 1152 | VOTED FOR THE CANDIDATE.

3. According to a recent 2013 government study using a 99% confidence interval, the average 4-year education at a public college or university will cost about $8,944 per year with a margin of error of ±7.2 %. (0.072) (8944) = 643.97



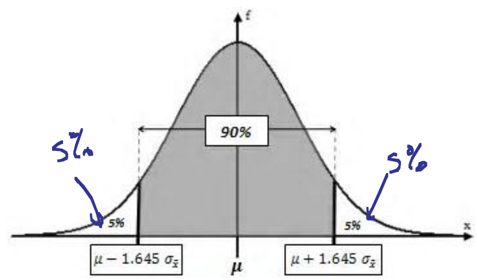
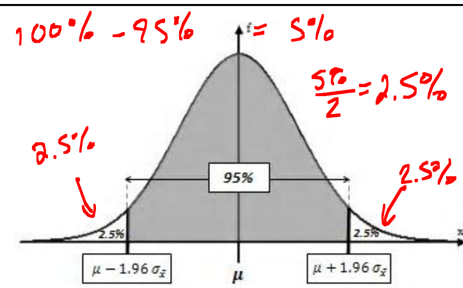
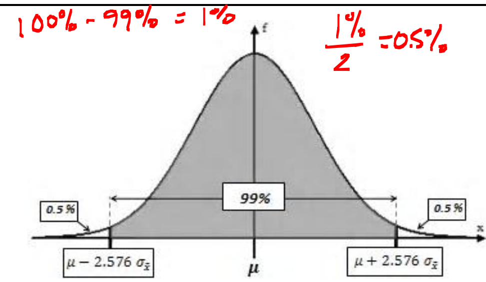
a. What is the minimum average cost of tuition?

$8944 - 643.97 = $8300.03

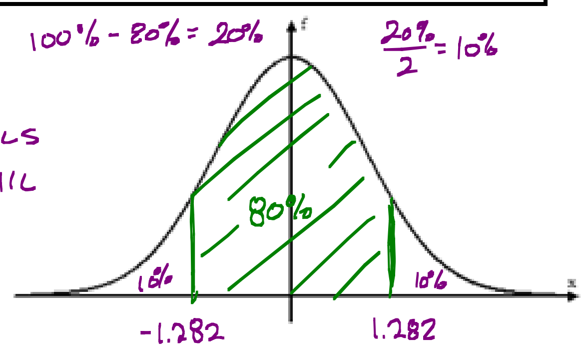
b. What is the ~~minimum~~ Maximum average cost of tuition?

$8944 + 643.97 = $9587.97

Most commonly confidence intervals of 90%, 95% and 99% are constructed. We will need to determine critical values for each of these which can be done with the "**InvNorm(**" command on your calculator.

Confidence Level	Critical value $\left(z_{\frac{\alpha}{2}}\right)$	Distribution
90%	**1.645**	
95%	**1.960**	
99%	**2.576**	

4. Using the "**InvNorm(**" command on your calculator determine the critical value for a confidence interval of 80%.



100% - 80% = 20% ← LEFT IN THE TAILS

20% ÷ 2 = 10% ← LEFT IN EACH TAIL

INV NORM(.10) ≈ -1.282

$\boxed{\text{CRITICAL VALUE} \approx 1.282}$

5. A random sample of 32 students about to graduate want to invite an average of 6 people to the graduation ceremony. If from previous studies the standard deviation has been determined to be 1.5 people, construct a interval estimate such that you would know with 99% confidence the mean number of invitations needed the graduates.

a. Determine the Standard Error

STANDARD ERROR : $\frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{32}} \approx 0.265$

b. Determine the Margin of Error.

MARGIN OF ERROR : (CRITICAL VALUE) $\sigma_{\bar{x}}$

: $(2.576)(0.265)$

≈ 0.683

c. Construct the Confidence Interval.

$6 - 0.683 < \mu < 6 + 0.683$

$\boxed{5.317 < \mu < 6.683}$

WITH 99% CONFIDENCE

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6-0.683        5.317
6+0.683        6.683
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6. In a random sample of 60 refrigerators, the mean repair cost was \$150.00 and the POPULATION standard deviation was \$15.50. Construct a 90% confidence interval for the population mean repair cost?

a. Determine the Standard Error

STANDARD ERROR : $\frac{\sigma}{\sqrt{n}} = \frac{15.50}{\sqrt{60}} \approx 2.001$

b. Determine the Margin of Error.

MARGIN OF ERROR : (CRITICAL VALUE) $\sigma_{\bar{x}}$

$\approx (1.645)(2.001)$

≈ 3.292

c. Construct the Confidence Interval.

$\$150 - 3.29 < \mu < \$150 + 3.29$

$\$146.71 < \mu < \$153.29$

$\boxed{\$146.71 < \mu < \$153.29}$

WITH 90% CONFIDENCE.

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150-3.29       146.71
150+3.29       153.29
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7. An admissions director wants to estimate the mean cost of books that each student must buy every year for a brochure she is creating. The estimate must be within \$10 and the costs are approximately normally distributed. The standard deviation of book prices is \$35

Determine the minimum required sample size to construct a 90% confidence interval for the population mean.

MARGIN OF ERROR = \$10

(CRITICAL VALUE) $\sigma_{\bar{x}}$ = 10

(CRITICAL VALUE) $\frac{\sigma}{\sqrt{n}} = 10$

$\frac{(1.645)}{1} \frac{35}{\sqrt{n}} = 10$

$\frac{57.575}{\sqrt{n}} = 10$

$\cancel{\sqrt{n}} \cdot \frac{57.575}{\cancel{\sqrt{n}}} = 10 \cdot \sqrt{n}$

$\frac{57.575}{10} = \frac{10 \cdot \sqrt{n}}{10}$

$(5.7575)^2 = (\sqrt{n})^2$

$33.15 \approx n$

34 PEOPLE IN THE SAMPLE

WOULD ENSURE $\pm$\$10 WITH 90% CONFIDENCE