

Trigonometric Identities

Reciprocal Identities:

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Co-function Identities:

$$\sin \theta = \cos(90^\circ - \theta) \quad \cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta) \quad \cot \theta = \tan(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta) \quad \csc \theta = \sec(90^\circ - \theta)$$

Opposite-Angle Identities:

$$\sin(-A) = -\sin A$$

$$\cos(-A) = \cos A$$

Sum & Difference Identities:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Double-Angle Identities:

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \end{aligned}$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Half-Angle Identities:

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}, \cos \alpha \neq -1$$

De Moivre's Theorem:

$$(\cos x + \sin x)^n = (\cos(nx) + \sin(nx))$$