Sec 4.1 – Trigonometric Identities

Basic Identities

Name:

Reciprocal Identities:

$\sin\theta = \dfrac{1}{\csc\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Using the Reciprocal and Quotient Identities **simplify** each as much as possible.

1. $tan(\theta) \cdot cos(\theta)$

2. $cot(\theta) \cdot cos(\theta) \cdot sin(\theta)$

3. $sin(\theta) \cdot cot(\theta) + cos(\theta)$

4. $csc(\theta) \cdot tan(\theta) - sec(\theta)$

5. $\dfrac{cot(\theta)}{csc(\theta)}$

6. $\dfrac{cot(\theta) \cdot sin(\theta)}{cos(\theta)} + cos(\theta) \cdot sec(\theta)$

7. $\dfrac{tan^2(\theta) \cdot csc(\theta) \cdot cos(\theta)}{sec(\theta)}$

Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Using the Reciprocal and Quotient Identities **verify** the following trigonometric identities.

8. $cot(\theta) \cdot sec(\theta) \cdot sin(\theta) = 1$

9. $\frac{cot(\theta)}{csc(\theta)} = cos(\theta)$

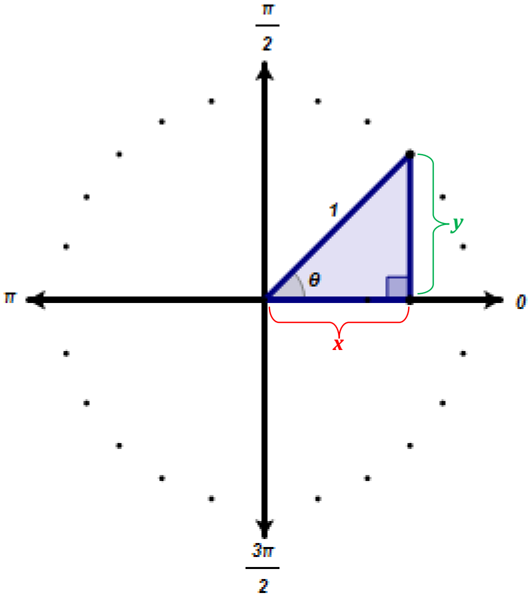
10. $\frac{sin(\theta)sec(\theta)}{tan(\theta)} + cos(\theta) \cdot sec(\theta) = 2$

11. $tan(\theta) + sec(\theta) = \frac{1+sin(\theta)}{cos(\theta)}$

12. $csc(\theta) + sec(\theta) = \frac{cos(\theta)+sin(\theta)}{cos(\theta)sin(\theta)}$

13. $\frac{1}{csc(\theta)} + tan(\theta) \cdot cos(\theta) = 2sin(\theta)$

Consider the following diagram.



- Using basic trigonometry solve for x in terms of θ.

- Using basic trigonometry solve for y in terms of θ.

- Write a Pythagorean Theorem statement using these expressions.

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

- Using the reciprocal identities verify $\tan^2\theta + 1 = \sec^2\theta$

- Using the reciprocal identities verify $1 + \cot^2\theta = \csc^2\theta$

Using the Reciprocal, Quotient, and Pythagorean Identities **simplify** each as much as possible.

14. $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

15. $\sin\theta\,(\sin\theta + \cos\theta\cot\theta)$

Reciprocal Identities		Quotient Identities	Pythagorean Identities
• $\sin\theta = \frac{1}{\csc\theta}$	• $\csc\theta = \frac{1}{\sin\theta}$	• $\tan\theta = \frac{\sin\theta}{\cos\theta}$	• $\sin^2\theta + \cos^2\theta = 1$
• $\cos\theta = \frac{1}{\sec\theta}$	• $\sec\theta = \frac{1}{\cos\theta}$	• $\cot\theta = \frac{\cos\theta}{\sin\theta}$	• $\tan^2\theta + 1 = \sec^2\theta$
• $\tan\theta = \frac{1}{\cot\theta}$	• $\cot\theta = \frac{1}{\tan\theta}$		• $1 + \cot^2\theta = \csc^2\theta$

Using the Reciprocal, Quotient, and Pythagorean Identities **simplify** each as much as possible.

16. $\sec\alpha - \sec\alpha\sin^2\alpha$

17. $\frac{\sin\theta cot\theta}{1-\sin^2\theta}$

18. $\frac{(\sec\theta+1)(\sec\theta-1)}{\sin^2\theta}$

19. $\frac{\csc x-\cos x\cot x}{\sin x}$

Reciprocal Identities

- $\sin\theta = \frac{1}{\csc\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\cos\theta = \frac{1}{\sec\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\tan\theta = \frac{1}{\cot\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Using the Reciprocal, Quotient, and Pythagorean Identities **verify** each trigonometric identity.

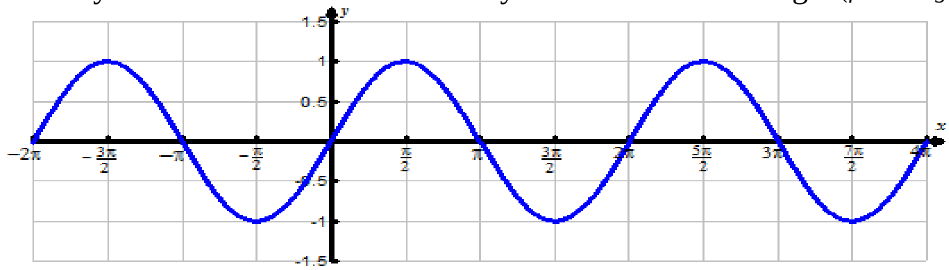
20. $\csc^2(x) - \csc^2(x)\cos^2(x) = 1$

21. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

22. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

23. $\frac{\tan\vartheta + \sin\vartheta}{1+\cos\vartheta} = \tan\vartheta$

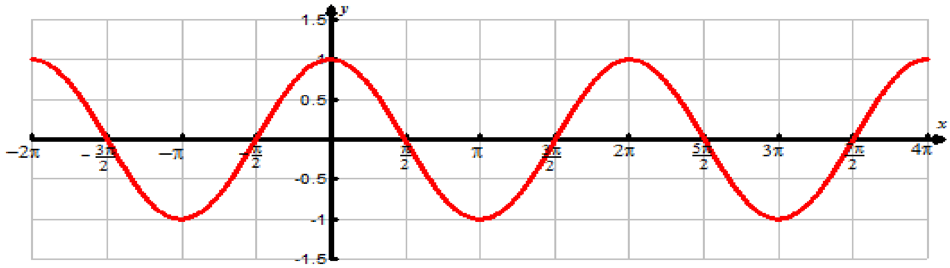
Consider that by definition an odd function is symmetric about the origin *(fold along the x-axis and the y-axis)*:



Therefore, if a function is odd we also know that $\boldsymbol{f(-x) = -f(x)}$ which would suggest

$$\mathbf{sin}(-x) = -\,\mathbf{sin}(x)$$

Consider that by definition an even function is symmetric about the y-axis *(fold along the y-axis)*:



Therefore, if a function is even we also know that $f(-x) = \boldsymbol{f(x)}$ which would suggest

$$\mathbf{cos}(-x) = \mathbf{cos}(x)$$

Even-Odd Identities:

$\sin(-x) = -\sin(x)$ $\qquad$ $\cos(-x) = \cos(x)$ $\qquad$ $\tan(-x) = -\tan(x)$

24. Verify: $\sec(x) = \frac{sin^2(x) + cos^2(x)}{\cos(-x)}$

25. Verify: $\csc(x) = \frac{sin^2(x) - 1}{\sin(-x)}$