Sec 4.1 – Trigonometric Identities

Basic Identities

Name:

Reciprocal Identities:

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad$ $\csc\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Using the Reciprocal and Quotient Identities **simplify** each as much as possible.

1. $tan(\theta)\cdot cos(\theta)$

$$\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos(\theta)}}{1} = \frac{\sin\theta}{1} = \sin(\theta)$$

2. $cot(\theta)\cdot cos(\theta)\cdot sin(\theta)$

$$\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cos\theta}{1}\cdot\frac{\cancel{\sin\theta}}{1} = \frac{\cos^2\theta}{1}$$

$$= \cos^2\theta$$

3. $sin(\theta)\cdot cot(\theta) + cos(\theta)$

$$\frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}} + \cos(\theta)$$

$$\cos(\theta) + \cos(\theta) = 2\cos(\theta)$$

4. $csc(\theta)\cdot tan(\theta) - sec(\theta)$

$$\underbrace{\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}} - \frac{1}{\cos\theta}$$

$$\frac{1}{\cos\theta} - \frac{1}{\cos\theta} = 0$$

5. $\dfrac{cot(\theta)}{csc(\theta)} = \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$$= \frac{\cos\theta}{\sin\theta}\div\frac{1}{\sin\theta} = \frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1} =$$

$$= \cos\theta$$

6. $\dfrac{cot(\theta)\cdot sin(\theta)}{cos(\theta)} + cos(\theta)\cdot sec(\theta)$

$$\frac{\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1}}{\frac{\cos\theta}{1}} + \frac{\cancel{\cos\theta}}{1}\cdot\frac{1}{\cancel{\cos\theta}}$$

$$\frac{\cos\theta}{\cos\theta} + 1$$

$$1 + 1 = 2$$

7. $\dfrac{tan^2(\theta)\cdot csc(\theta)\cdot cos(\theta)}{sec(\theta)} = \dfrac{\frac{\sin\theta\,\cancel{\sin\theta}}{\cos\theta\,\cancel{\cos\theta}}\cdot\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\cos\theta}}{1}}{\frac{1}{\cos\theta}} = \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$$= \frac{\sin\theta}{\cos\theta}\div\frac{1}{\cos\theta} = \frac{\sin\theta}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{1} = \boxed{\sin(\theta)}$$

Reciprocal Identities:

$\sin\theta = \dfrac{1}{\csc\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$
$\cos\theta = \dfrac{1}{\sec\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$
$\tan\theta = \dfrac{1}{\cot\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Using the Reciprocal and Quotient Identities **<u>verify</u>** the following trigonometric identities.

8. $cot(\theta) \cdot sec(\theta) \cdot sin(\theta) = 1$

$$\frac{\cancel{\cos(\theta)}}{\cancel{\sin(\theta)}} \cdot \frac{1}{\cancel{\cos(\theta)}} \cdot \frac{\cancel{\sin(\theta)}}{1} = 1$$

$$\frac{1}{1} = 1$$

$$1 = 1 \checkmark$$

9. $\dfrac{cot(\theta)}{csc(\theta)} = cos(\theta)$

$$\frac{\;\frac{\cos\theta}{\sin\theta}\;}{\frac{1}{\sin\theta}} = \cos(\theta)$$

$$\frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin\theta} = \cos(\theta)$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \cos\theta$$

$$\cos\theta = \cos\theta \checkmark$$

10. $\dfrac{sin(\theta)sec(\theta)}{tan(\theta)} + cos(\theta) \cdot sec(\theta) = 2$

$$\frac{\frac{\sin\theta}{1}\,\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta}{1} \cdot \frac{1}{\cos\theta} = 2$$

$$\frac{\cancel{\frac{\sin\theta}{\cos\theta}}}{\cancel{\frac{\sin\theta}{\cos\theta}}} + \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}} = 2$$

$$1 + 1 = 2$$

$$2 = 2 \checkmark$$

11. $tan(\theta) + sec(\theta) = \dfrac{1+sin(\theta)}{cos(\theta)}$

$$\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta + 1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \checkmark$$

12. $csc(\theta) + sec(\theta) = \dfrac{cos(\theta)+sin(\theta)}{cos(\theta)sin(\theta)}$

$$\frac{\cos\theta \cdot 1}{\cos\theta \cdot \sin\theta} + \frac{1 \cdot \sin\theta}{\cos\theta \cdot \sin\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta}$$

$$\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta}$$

$$\frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta} \checkmark$$

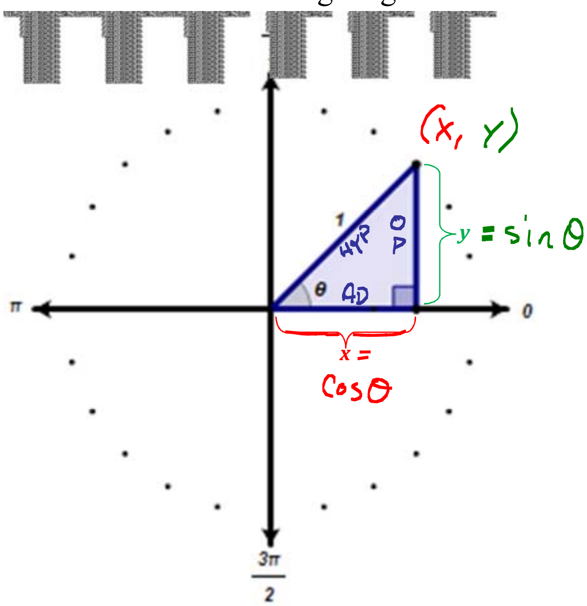
13. $\dfrac{1}{csc(\theta)} + tan(\theta) \cdot cos(\theta) = 2sin(\theta)$

$$\sin\theta + \left(\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}\right) = 2\sin(\theta)$$

$$\underbrace{\sin\theta + \sin\theta} = 2\sin(\theta)$$

$$2\sin\theta = 2\sin(\theta) \checkmark$$

Consider the following diagram.



- Using basic trigonometry solve for x in terms of θ.

$$\cos(\theta) = \frac{AD}{HYP} = \frac{x}{1}$$

$$\cos(\theta) = x$$

- Using basic trigonometry solve for y in terms of θ.

$$\sin(\theta) = \frac{OP}{HY} = \frac{y}{1}$$

$$\sin(\theta) = y$$

- Write a Pythagorean Theorem statement using these expressions.

$$a^2 + b^2 = c^2$$

$$\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

- Using the reciprocal identities verify $\tan^2\theta + 1 = \sec^2\theta$

$$\frac{1}{\cos^2\theta}\left(\sin^2\theta + \cos^2\theta\right) = (1)\cdot\frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

- Using the reciprocal identities verify $1 + \cot^2\theta = \csc^2\theta$

$$\frac{1}{\sin^2\theta}\left(\sin^2(\theta) + \cos^2(\theta) = 1\right)$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

Using the Reciprocal, Quotient, and Pythagorean Identities **simplify** each as much as possible.

14. $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

15. $\sin\theta\,(\sin\theta + \cos\theta\cot\theta)$

$$\sin\theta\cdot\sin\theta + \cos\theta\cot\theta\cdot\sin\theta$$

$$\sin^2\theta + \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1}$$

$$\sin^2\theta + \cos^2\theta$$

$$1$$

Reciprocal Identities		Quotient Identities	Pythagorean Identities
• $\sin\theta = \frac{1}{\csc\theta}$	• $\csc\theta = \frac{1}{\sin\theta}$	• $\tan\theta = \frac{\sin\theta}{\cos\theta}$	• $\sin^2\theta + \cos^2\theta = 1$
• $\cos\theta = \frac{1}{\sec\theta}$	• $\sec\theta = \frac{1}{\cos\theta}$	• $\cot\theta = \frac{\cos\theta}{\sin\theta}$	• $\tan^2\theta + 1 = \sec^2\theta$ $-1 \quad\quad -1$
• $\tan\theta = \frac{1}{\cot\theta}$	• $\cot\theta = \frac{1}{\tan\theta}$		• $1 + \cot^2\theta = \csc^2\theta$

Using the Reciprocal, Quotient, and Pythagorean Identities **simplify** each as much as possible.

16. $\sec\alpha - \sec\alpha\sin^2\alpha$

$= \frac{1}{\cos\alpha} - \frac{1}{\cos\alpha}\cdot\frac{\sin^2\alpha}{1}$

$= \frac{1}{\cos\alpha} - \frac{\sin^2\alpha}{\cos\alpha}$

$= \frac{1-\sin^2\alpha}{\cos\alpha}$

$= \frac{\cancel{\cos\alpha}\,\cos\alpha}{\cancel{\cos\alpha}}$

$= \cos\alpha$

* $\sin^2\theta + \cos^2\theta = 1$
$-\sin^2\theta \quad\quad -\sin^2\theta$

* $\cos^2\theta = 1-\sin^2\theta$

ALTERNATE FORMS
- $\cos^2\theta = 1-\sin^2\theta$
- $\sin^2\theta = 1-\cos^2\theta$
- $\sin^2\theta + \cos^2\theta = 1$

17. $\frac{\sin\theta\cot\theta}{1-\sin^2\theta}$

$= \frac{\frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}}}{\cos^2\theta}$

$= \frac{\cos\theta}{\cos^2\theta} = \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}\,\cos\theta} = \frac{1}{\cos\theta}$

$= \sec\theta$

18. $\frac{(\sec\theta+1)(\sec\theta-1)}{\sin^2\theta}$

$\frac{\sec^2\theta - \cancel{\sec\theta} + \cancel{\sec\theta} - 1}{\sin^2\theta}$

$\frac{\sec^2\theta - 1}{\sin^2\theta}$

* $\tan^2\theta + 1 = \sec^2\theta$
$-1 \quad\quad -1$
$\tan^2\theta = \sec^2\theta - 1$

$\frac{\tan^2\theta}{\sin^2\theta}$

$\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sin^2\theta} = \frac{\cancel{\sin^2\theta}}{\cos^2\theta}\cdot\frac{1}{\cancel{\sin^2\theta}}$

$\frac{1}{\cos^2\theta} = \boxed{\sec^2\theta}$

19. $\frac{\csc x - \cos x\cot x}{\sin x}$

$= \frac{\frac{1}{\sin x} - \frac{\cos x}{1}\cdot\frac{\cos x}{\sin x}}{\sin x} = \frac{\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}}{\sin x}$

$= \frac{1-\cos^2 x}{\sin x} \div \frac{\sin x}{1}$

$= \frac{\sin^2 x}{\sin x}\cdot\frac{1}{\sin x}$

$= \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}} = \boxed{1}$

Reciprocal Identities		**Quotient Identities**	**Pythagorean Identities**
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\tan^2\theta + 1 = \sec^2\theta$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$		$1 + \cot^2\theta = \csc^2\theta$

Using the Reciprocal, Quotient, and Pythagorean Identities **verify** each trigonometric identity.

20. $\csc^2(x) - \csc^2(x)\cos^2(x) = 1$

$$\frac{1}{\sin^2(x)} - \frac{1}{\sin^2(x)}\cdot\frac{\cos^2(x)}{1} = 1$$

$$\frac{1}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} = 1$$

$$\frac{1-\cos^2(x)}{\sin^2(x)} = 1$$

$$\frac{\sin^2(x)}{\sin^2(x)} = 1$$

$$1 = 1 \checkmark$$

* ALT PYTHAG

$s^2 + c^2 = 1$

$s^2 = 1 - c^2$

$c^2 = 1 - s^2$

21. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$$\sin\theta\cot\theta + \sin\theta\tan\theta = \sec\theta$$

$$\frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}} + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} = \sec\theta$$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta} = \sec\theta$$

$$\frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} = \sec\theta$$

$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \sec\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\sec\theta = \sec\theta \checkmark$$

22. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} = \sec\theta\csc\theta$$

$$\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} = \sec\theta\csc\theta$$

$$\sec\theta\cdot\csc\theta = \sec\theta\csc\theta \checkmark$$

23. $\dfrac{\tan\vartheta + \sin\vartheta}{1+\cos\vartheta} = \tan\vartheta$

$$\frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}}{1+\cos\theta} = \tan\theta$$

$$\frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\cos\theta}}{1+\cos\theta} = \tan\theta$$

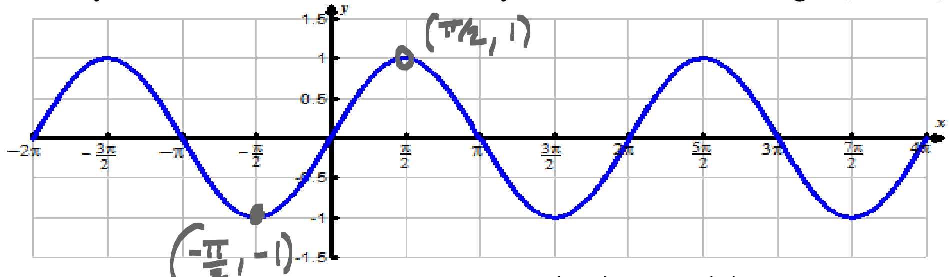
$$\frac{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}}{1+\cos\theta} = \tan\theta$$

$$\frac{\sin\theta\cancel{(1+\cos\theta)}}{\cos\theta}\cdot\frac{1}{\cancel{(1+\cos\theta)}} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \tan\theta \checkmark$$

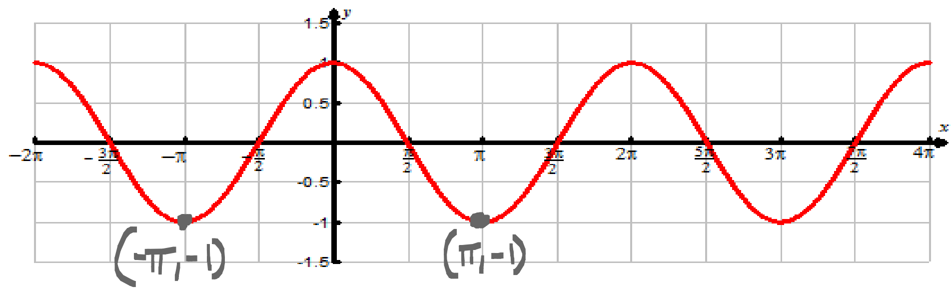
Consider that by definition an odd function is symmetric about the origin *(fold along the x-axis and the y-axis)*:



Therefore, if a function is odd we also know that $\boldsymbol{f(-x) = -f(x)}$ which would suggest

$\boxed{\sin(-x) = -\sin(x)}$

Consider that by definition an even function is symmetric about the y-axis *(fold along the y-axis)*:



Therefore, if a function is even we also know that $\boldsymbol{f(-x) = f(x)}$ which would suggest

$\cos(-x) = \cos(x)$

Even-Odd Identities:

$\sin(-x) = -\sin(x)$ $\qquad \cos(-x) = \cos(x)$ $\qquad \tan(-x) = -\tan(x)$

24. Verify: $\sec(x) = \dfrac{\sin^2(x) + \cos^2(x)}{\cos(-x)}$

$\sec(x) = \dfrac{1}{\cos(x)}$

$\sec(x) = \sec(x)$ ✓

25. Verify: $\csc(x) = \dfrac{\sin^2(x) - 1}{\sin(-x)}$

$\csc(x) = \dfrac{-\cos^2(x)}{-\sin(x)}$

$\csc(x) = \dfrac{\cos(x)\cos(x)}{\sin(x)}$

$\csc(x) = \dfrac{\cos(x)}{\sin(x)} \cdot \dfrac{\cos(x)}{1}$

$\csc(x) = \cot(x) \cdot \cos(x)$ X

ALT IDs

$s^2 + c^2 = 1$

$s^2 = 1 - c^2$

$c^2 = 1 - s^2$

$s^2 = 1 - c^2$
$-1 \quad -1$

$s^2 - 1 = -c^2$

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Plot1 Plot2 Plot3
\Y1■1/sin(X)
\Y2■((sin(X))²-1
)/sin(-X)
\Y3=
\Y4=
\Y5=
\Y6=
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NOT AN IDENTITY

← DOES NOT MATCH