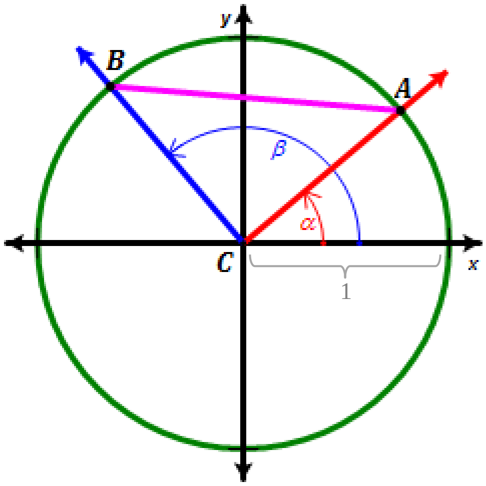
Sec 4.2 – Trigonometric Identities
Sum & Difference Identities

Name:



Consider the diagram at the right of a unit circle.

1. First, determine the coordinates of point A in terms of α.

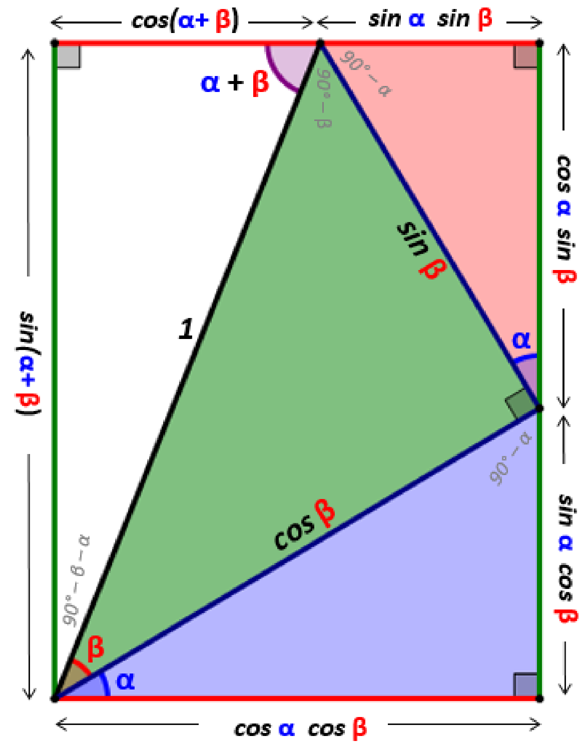
2. First, determine the coordinates of point B in terms of β.

3. Using those coordinates and the distance formula, find the distance between AB in terms of α and β.

4. Using the Law of Cosines and triangle ABC, find the length of side AB in terms of α and β.

5. Use the two unique descriptions of the length of AB create a new trigonometric identity.

Alternately, you could use the following elegant diagram to show the sum identities hold true for angles from 0° to 90° or 0 to $\frac{\pi}{2}$ by realizing opposite sides of a rectangle must have the same measure. Further, the difference identities can be determined by replacing β with negative β and simplifying.

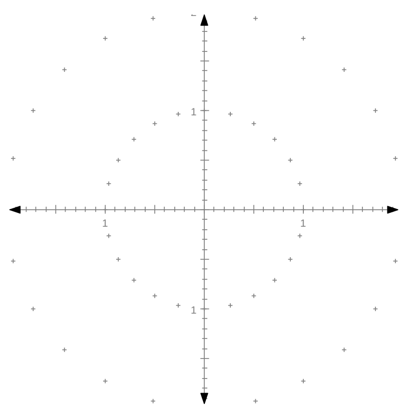


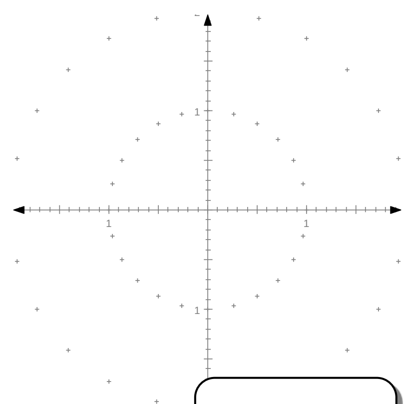
$$sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

1. Find the **exact** value of $\sin(75°)$

(using sum and difference identities)

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

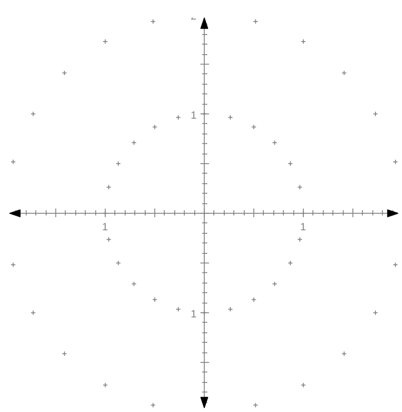


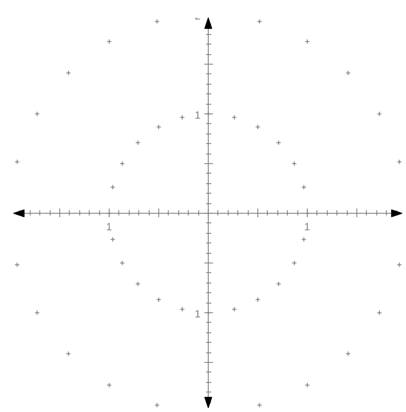


2. Find the **exact** value of $\cos(255°)$

(using sum and difference identities)

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

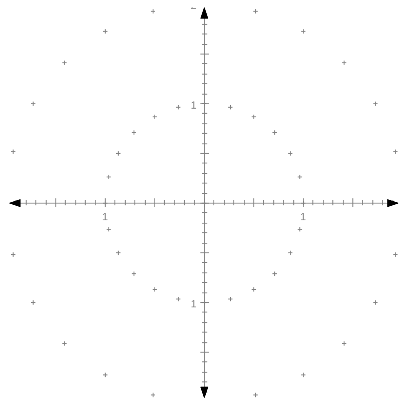


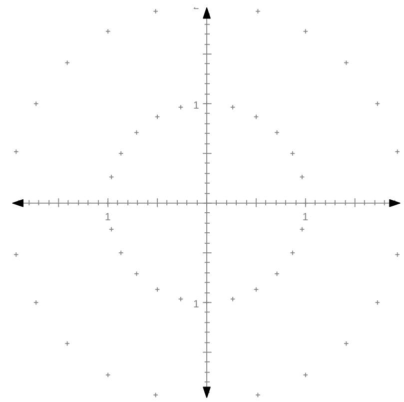


3. Find the **exact** value of $\sin\left(\frac{13\pi}{12}\right)$

(using sum and difference identities)

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

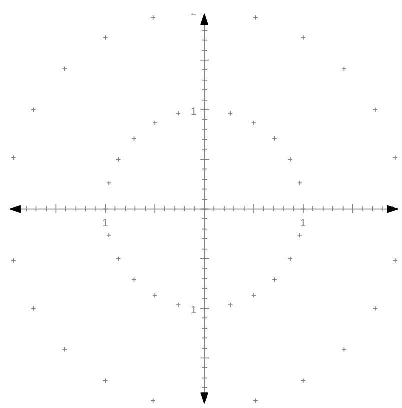


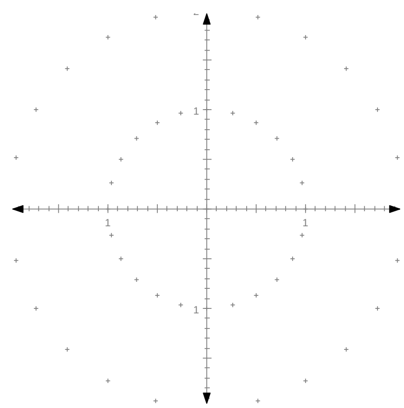


4. Find the **exact** value of $\cos\left(\frac{5\pi}{12}\right)$

(using sum and difference identities)

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

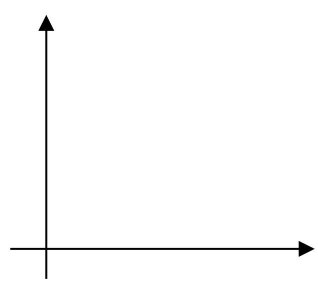


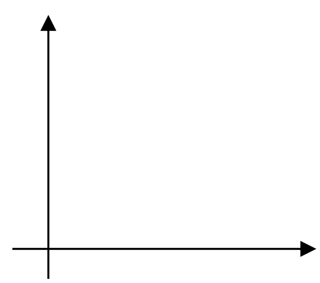


5. Given that $\sin(A) = \frac{5}{13}$ and $\cos(B) = \frac{3}{5}$. Also, assume A & B are in the first quadrant.

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

Find the **exact** value of $\sin(A - B) =$

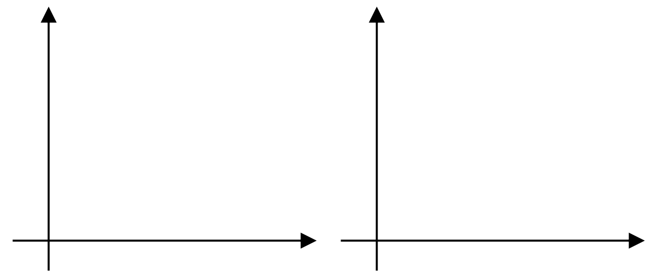




6. Given that $\cos(A) = \frac{9}{13}$ and $\cos(B) = \frac{1}{5}$. Also, assume A & B are in the first quadrant.

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Find the **exact** value of $\cos(A+B) =$



Simplify the following trigonometric expressions using the sum and difference identities.

7. $sin(\pi + \theta)$

8. $cos(\theta + \theta)$

Determine the sum identity for tangent using the sum identities for sine and cosine.

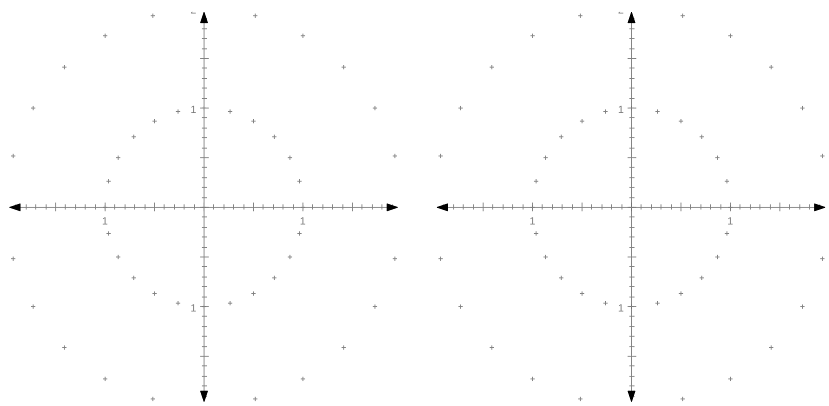
$$\text{Tan}(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta} \cdot \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$$

$$\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} = 1$$

9. Find the **exact** value of $\tan(255°)$

(using sum and difference identities)

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$



10. Given that $\sin(A) = \frac{5}{13}$ and $\cos(B) = \frac{8}{17}$. Also, assume A & B are in the first quadrant.

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Find the **exact** value of $\tan(A - B) =$