Sec 4.3 – Trigonometric Identities
Half Angle Identities

Name:

Starting with the double angle identities, create the half angle identities:

Double-Angle Identities

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\cos(2\theta) = 1 - 2\sin^2\theta$
- $\cos(2\theta) = 2\cos^2\theta - 1$
- $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

1. Let $\theta = \frac{\alpha}{2}$ using the identity
   $\cos(2\theta) = 1 - 2\sin^2\theta$
   and solve for $\sin\frac{\theta}{2}$

2. Let $\theta = \frac{\alpha}{2}$ using the identity
   $\cos(2\theta) = 2\cos^2\theta - 1$
   and solve for $\cos\frac{\theta}{2}$

3. Use the two identities you just created in problems 1 and 2, simplify the following to create the tangent half angle identity.

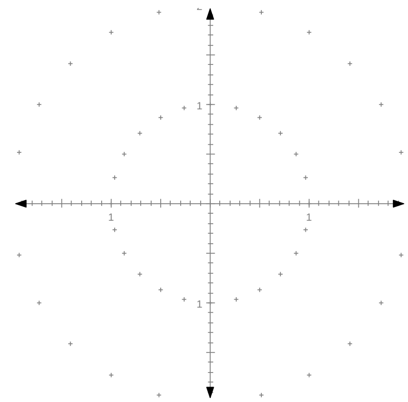
$$\tan\frac{\theta}{2} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} =$$

Half-Angle Identities

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

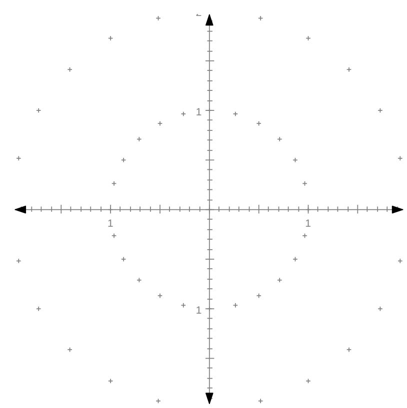
4. Find the **exact** value of $\cos(112.5°)$

(using half angle identities)



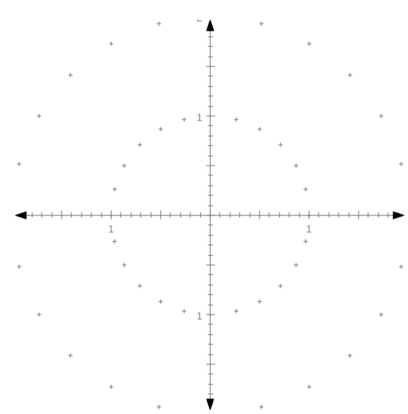
5. Find the **exact** value of $\sin\left(\frac{\pi}{12}\right)$

(using half angle identities)

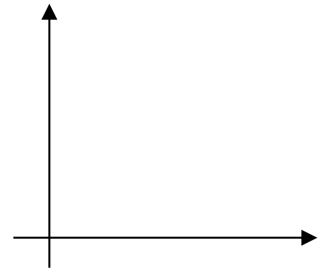


6. Find the **exact** value of $\tan(15°)$

(using half angle identities)



7. Given that $\sin(\alpha) = \frac{5}{13}$ and assume angle α is in the first quadrant.



a. Find the **exact** value of $\sin\left(\frac{\alpha}{2}\right) =$

b. Find the **exact** value of $\cos\left(\frac{\alpha}{2}\right) =$

c. Find the **exact** value of $\tan\left(\frac{\alpha}{2}\right) =$

8. Verify the following identities using the half angle identities.

a. $2 - 2\cos^2\left(\frac{\theta}{2}\right) = 1 - \cos\theta$

b. $\frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$