Adv. Mathematical Decision Making

Sec 8.6 – Markov Chains
Voting methods & Matrix Applications Name:

A Markov chain is a process that arises naturally in problems that involve a finite number of events or states that change over time.

A company is doing a research study in the metro-Atlanta area about smart phones (either android or iPhone) and determined that 56% of the sample owned iPhones and the other 44% owned Android based phones. After following the customers for a while, the researcher determined that each year after that 80% of the iPhone owners purchased a new iPhone again when it came time to renew their contract. The remaining 20% switched to Android. The researcher also determined that 65% of android owners purchased a new Android phone again when it came time to renew their contract. The remaining 35% of android owners switched to an iPhone.

a) Create a tree diagram showing the percentages of each owner after one cycle of renewals.

iPhone (.56)
- iPhone (.80) → 0.448
- ANDROID (.20) → 0.112

ANDROID (.44)
- iPhone (.35) → 0.154
- ANDROID (.65) → 0.286

NEW iPhone PERCENT: .448 + .154 = .602

NEW ANDROID PERCENT: .112 + .286 = .398

b) Create the initial distribution matrix D_o

$$[D_o] = \begin{array}{cc} \text{iPhone} & \text{ANDROID} \\ \end{array}$$
$$[D_o] = \begin{bmatrix} .56 & .44 \end{bmatrix}$$

c) Create a Transition Matrix.

$$[T] = \begin{array}{c|cc} & \text{iPhone} & \text{ANDROID} \\ \hline \text{FROM iPhone} & 0.80 & 0.20 \\ \text{ANDROID} & 0.35 & 0.65 \end{array}$$

(TO: iPhone, ANDROID)

d) In the study there were initially 224 iPhone owners and 176 Android owners. If each participant purchases a new phone every 2 years, determine how many of the participants would own iPhones and how many would own Androids 8 years after the study began?

$$\begin{bmatrix} 224 & 176 \end{bmatrix} \cdot \begin{bmatrix} .8 & .2 \\ .35 & .65 \end{bmatrix}^4 \approx \begin{bmatrix} 253 & 147 \end{bmatrix}$$

(iPhone, ANDROID)

Suppose that in the Northwest region of the country, Ford dealers make up 40% of the automobile sales, GM dealers make up 45% and foreign car dealers 15%. The foreign car dealers decide to offer a no-interest-for-6-months-after-purchase incentive plan and during the next year of business, the following transition matrix evolves:

		TO		
		Ford	GM	Foreign
FROM	Ford	0.50	0.20	0.30
	GM	0.30	0.40	0.30
	Foreign	0.15	0.10	0.75

Based on this matrix, what percentage of GM drivers will switch to Ford each year?

30%

After year 3, what percentage of drivers will be driving Fords?

$$\begin{array}{ccc} \text{Ford} & \text{GM} & \text{F} \end{array}$$
$$\begin{bmatrix} .4 & .45 & .15 \end{bmatrix} \cdot \begin{bmatrix} .5 & .2 & .3 \\ .3 & .4 & .3 \\ .15 & .1 & .75 \end{bmatrix}^3 = \begin{bmatrix} .293 & .197 & .509 \end{bmatrix}$$

$\boxed{\approx 29.3\%}$

What percentage of drivers will be driving Fords long-term?

$\boxed{\approx 27.3\%}$

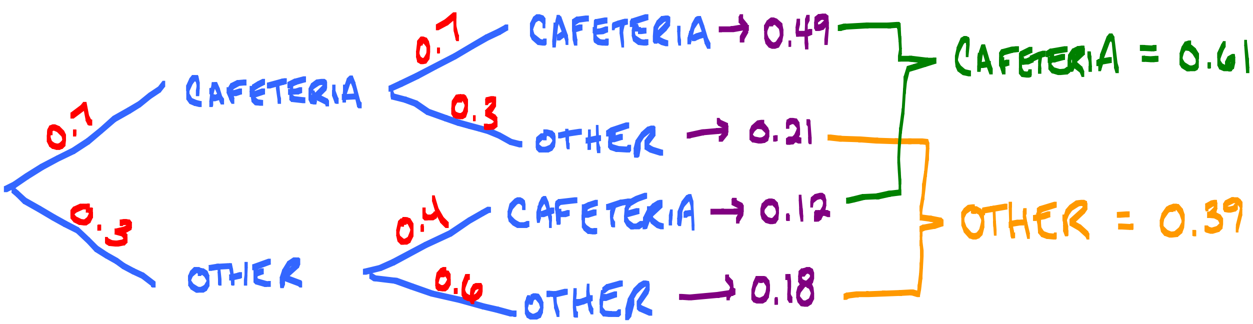
A food service director for a local high school conducted a survey in hopes of predicting the number of students who will eat in the cafeteria in the future. The results of the survey are as follows:

- If a student eats in the cafeteria on a given day, the probability that he or she will eat there again the next day is 70% and the probability that he or she will not eat there is 30%.

- If a student does not eat in the cafeteria on a given day, the probability that he or she will eat in the cafeteria the next day is 40% and the probability that he or she will not eat there is 60%.

On Monday, 70% ate at the cafeteria and 30% at somewhere else

a) Create a tree diagram showing the percentages of each owner after one cycle of renewals.



b) Create the initial distribution matrix D_o

$$[D_o] = \begin{array}{cc} \text{CAFETERIA} & \text{OTHER} \\ \end{array} \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}$$

c) Create a Transition Matrix.

$$[T] = \begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{TO}} \\ & & \text{CAFETERIA} & \text{OTHER} \\ \text{FROM} & \text{CAFETERIA} & 0.7 & 0.3 \\ & \text{OTHER} & 0.4 & 0.6 \end{array}$$

d) Find D_3

$$[D_3] = [D_o] \cdot [T]^3$$

$$= \begin{bmatrix} .5749 & .4251 \end{bmatrix}$$

e) How many students could be expected to eat at the cafeteria on Friday if there were 350 students that ate at the cafeteria and 150 that did not on Monday?

$$\begin{bmatrix} 350 & 150 \end{bmatrix} \cdot [T]^4 \approx \begin{bmatrix} 286 & 214 \end{bmatrix}$$

CAFETERIA OTHER