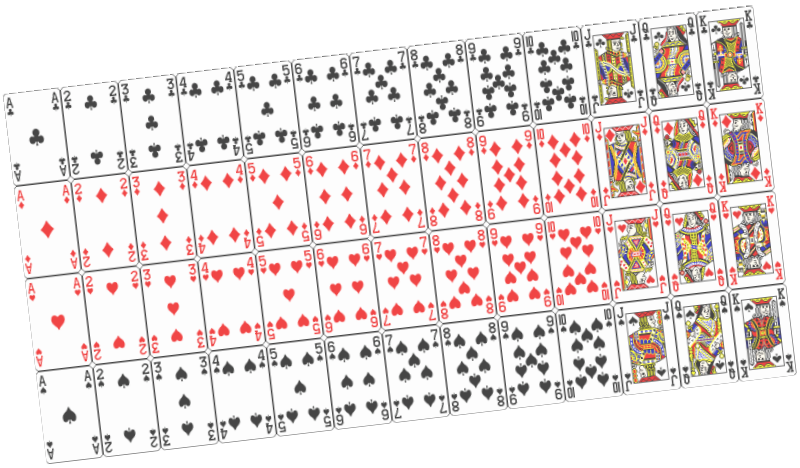
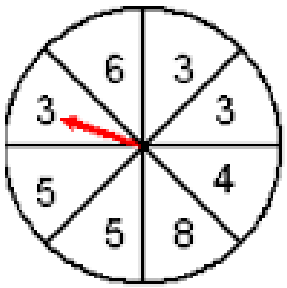
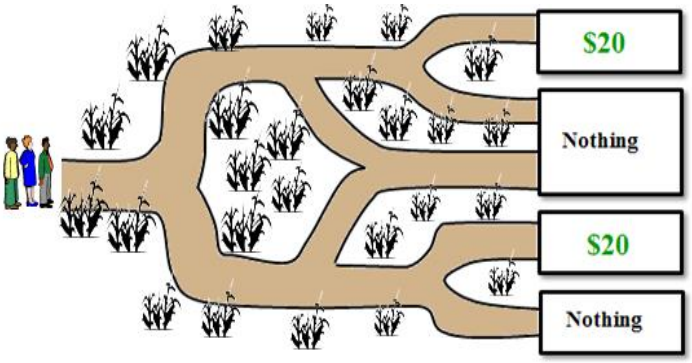
Adv. Math Decision Making

Unit 2:
Using Probability







MAtTHew M. Winking

Sec 2.1 -Probability
Basic Probability

Name: _____

Determine whether each of the following are <u>EMPIRICAL</u> ,<u>THEORETRICAL</u> ,or <u>SUBJECTIVE</u> probability and then determine the probability.

1. What is the probability of rolling a standard six sided number cube to a number with 4 letters in the spelling of that number?

Circle one of the following:		
Theoretical	Empirical	Subjective

Fraction:	Decimal:	Percent:

2. Janice attempted to shoot 34 free throws with a basketball. She made 19 of the free throws. Based on those results what would you estimate the probability to be that she makes the next free throw?

Circle one of the following:		
Theoretical	Empirical	Subjective

Fraction:	Decimal:	Percent:

3. A weather forecaster suggests that there is a 40% chance of snow and ice on the next day.

Circle one of the following:		
Theoretical	Empirical	Subjective

Fraction:	Decimal:	Percent:

4. A person has 5 coins in their pocket. A dime, 2 pennies, a quarter, and a nickel. If a person randomly picks one coin out of their pocket. What would the probability be that they get a penny?

Circle one of the following:		
Theoretical	Empirical	Subjective

Fraction:	Decimal:	Percent:

5. Define ***<u>Complement</u>*** *of an event*: _____

_____

6. What is the ***<u>Complement</u>*** of drawing a "numbered" card from a standard deck of shuffled cards? _____

_____

7. What is the difference between the **<u>probability</u>** and the **<u>odds</u>** of winning? _____

_____

8. Determine the following.
   A. What are the **<u>odds</u>** of rolling a standard six sided die to the number "2"?
   B. If the probability of winning is 20% what are the **<u>odds</u>** of winning?

CLASSIC PROBABILITY

9. What is the probability of flipping a coin and having it land on "Tails"? (write your answer in fractional form, decimal form, and as a percentage)

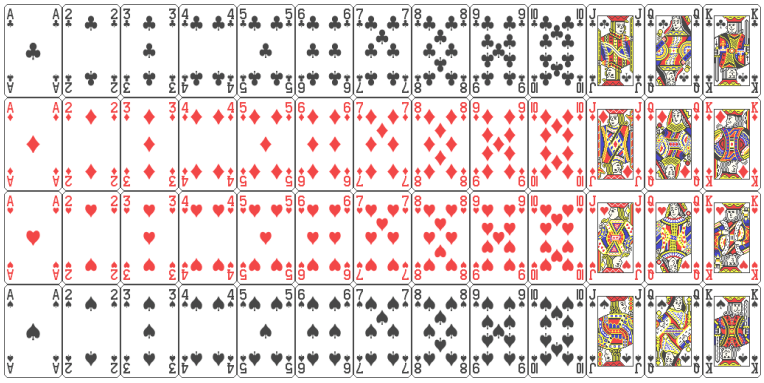
Fraction:	*Decimal:*	*Percent:*

10. What is the probability of drawing an odd numbered card from a standard deck of shuffled cards?

Decimal:

11. A person has 20 Rap songs, 15 R&B songs, 20 Alternative songs, 12 Pop songs, 5 Country songs, and 18 Hip Hop songs stored on an MP3 player (a total of 90 songs). If the person set the MP3 player to random shuffle, what are the odds of the first song being played being a Country song?

Odds:



12. What is the probability of flipping two coins and having both coins land on tails *(make a tree diagram first)*?

Fraction:

13. What is the probability of flipping 3 coins and having al l three land on tails *(make a tree diagram first)*?

Fraction:

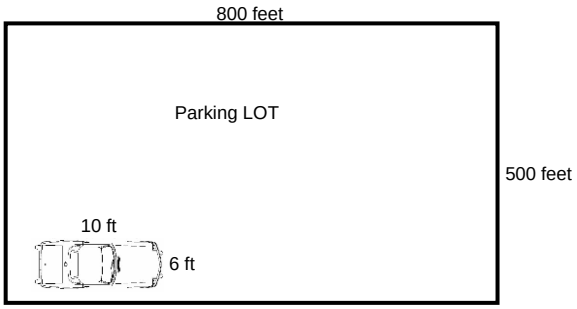
14. What is the probability of flipping 3 coins and having exactly two of them land on tails?

Fraction:

GEOMETRIC and CONTINUOUS PROBABILITIES

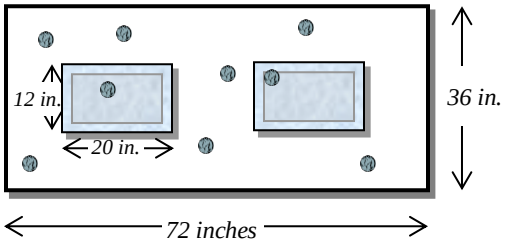
15. Determine the following **geometric** probabilities:

A. Tal-mart parking lot is 500 feet by 800 feet. You just parked your car, which is approximately a rectangle with the dimensions 6 feet by 10 feet. A bird flying overhead has just relieved himself by having a bowel movement. If the waste material has an equal chance of landing anywhere in the parking lot then what are the chances of it hitting your car.



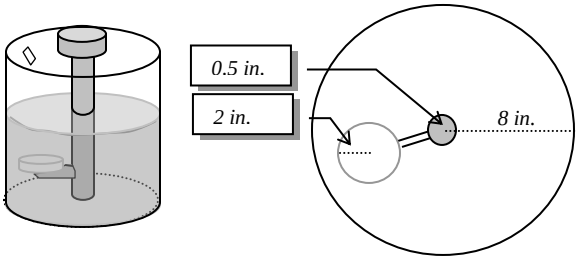
Decimal:

B. At an amusement park they allow attendants to try and throw a quarter onto one of two rectangular glass plates from a distance. The glass plates are sitting on a rectangular table. If the quarter is returned to the thrower if it misses the table and we are assuming it equally likely to land anywhere on the table, then what is the probability of landing the quarter somewhere on one of the two glass plates?



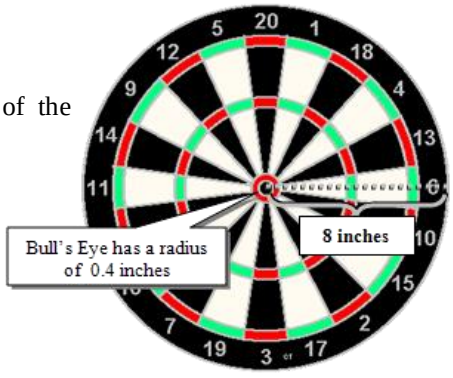
Percent:

C. At a Taco Mel's they have a game in which you drop a dime in an aquarium type device. If the dime lands on a special platform you win a free meal. Determine your odds of winning the free meal if it looks like the diagram below.



Decimal:

D. Assuming a dart is to land somewhere on a dart board, what is the probability of the dart hitting the bull's eye?



Decimal:

16. If a cable repairman is supposed to show up to for a repair between 1:00pm and 6:00pm and the home owner can't be home until 3:00pm, what is the probability that the home owner will be home when the cable repair man shows up (assuming the repairman is equally likely to come at any time during the specified range)?

Percentage:

Sec 2.2 -Probability
Independent & Dependent Probabilities Name:

INDEPENDENT PROBABILITY

1. Determine the following probabilities if each of the following are **independent**.

GIVEN:	**P(A) = 0.8**	**P(B) = 0.25**	**P(C) = 0.6**

a. P(A and C) = *Decimal:*

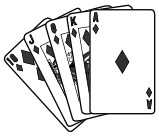
b. P(A and B and C) = *Decimal:*

c. P(Rolling a 4 on a standard die and B) =

Decimal:

d. Find the P(D) assuming A and D are independent events, and given P(A and D) = 0.24

Decimal:



e. P(Rolling a 2 on a standard die and picking a card with a "7" on it from a standard deck of cards) =

Decimal:

f. If you were to roll two standard six-sided dice, what is the probability that both land on a 5 or greater?

Decimal:

g. If your chances of losing the shell game if you randomly pick is 2 in 3. What are the chances that you would lose 5 games in a row?

Decimal:

h. Assuming a baseball Freddie has a batting average of 0.305. If we assume the probability of him getting a base hit is 0.305, what is the probability that he gets 3 base hits in a row?

Decimal:

i. A weather forecaster suggests the probability of rain on Saturday is 30% and on Sunday there is a 60% chance of rain. If the forecaster's probabilities are correct what is the probability of no rain all weekend?

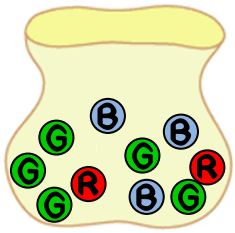
Decimal:

j. ** Nancy estimates that the probability that a tornado will strike with in the city limits on any given year is 0.75%. What is the probability of at least one tornado touching down in the next 5 years? *(must use complements)*

Decimal:

DEPENDENT PROBABILITIES

2. Consider an opaque bag with 5 green marbles, 3 blue marbles, and 2 red marbles.



   a. If two marbles are drawn without replacement what is the probability that they are both green?

   Decimal:

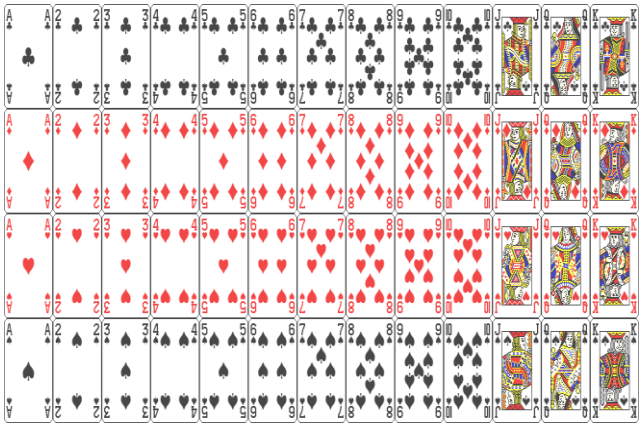
   b. If two marbles are drawn without replacement what is the probability that the first draw is a red marble and the second draw is a blue marble?

   Decimal:

   c. ***If two marbles are drawn without replacement what is the probability that the both marbles are the same color?

   Decimal:

3. Consider that 3 consecutive cards are drawn without replacement from a shuffled deck of cards (**Dependent probability**)



   A. What is the probability that the first two cards drawn are face cards?

   Decimal:

   B. What is the probability that the all three cards are hearts?

   Decimal:

   C. What is the probability that all three cards are a King?

   Decimal:

   D. What is the probability that all three cards are the same (BONUS)?

   Decimal:

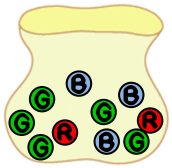
Sec 2.3 -Probability
Conditional Probability & Exclusive Events

Name:

CONDITIONAL PROBABILITY

1. Determine the following **conditional** probabilities.

   Consider a bag with marbles, 3 blue marbles, 2 red marbles, and 5 green marbles. Three marbles are drawn in sequence and are taken without replacement.



   i. **P(**2nd draw: **blue** | 1st draw: **red)** = *Reduced Fraction:*

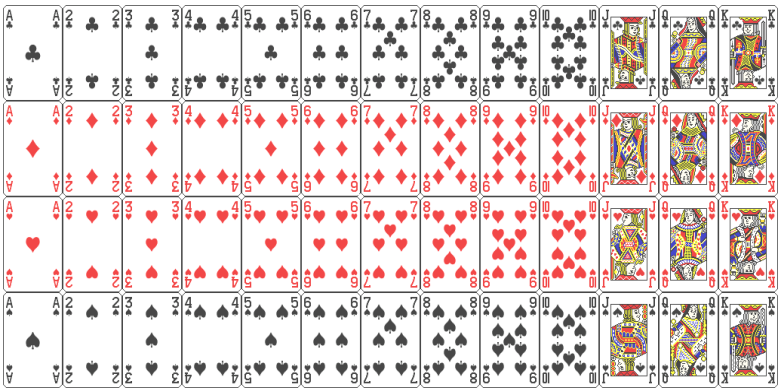
   ii. **P(**2nd draw: **blue** | 1st draw: **blue)** = *Reduced Fraction:*

   iii. **P(**3rd draw: **blue** | 1st draw: **red,** 2nd draw: **blue)** = *Reduced Fraction:*

   ii. **P(**1st draw: **blue** | 1st draw: **red)** = *Reduced Fraction:*

2. Determine the following **conditional** probabilities.

   Consider drawing 1 card from a standard deck of shuffled cards:



   i. P(Queen | Face Card) = *Reduced Fraction:*

   ii. P(Heart with a Number | Red Card) = *Reduced Fraction:*

   iii. P(Card with a Letter| King) = *Reduced Fraction:*

   iv. P(number less than 6 | Face Card) = *Reduced Fraction:*

   v. P(Black Jack | Face Card) = *Reduced Fraction:*

3. Consider the following table with information about all of the students taking Statistics at Phoenix High School.

	Full-time	Part-Time	Total
Female	28	15	43
Male	12	16	28
Total	40	31	71

   A. P(Full-time | Male) = *Reduced Fraction:*

   B. P(Male | Full-time) = *Reduced Fraction:*

   C. P(Female | Part-time) = *Reduced Fraction:*

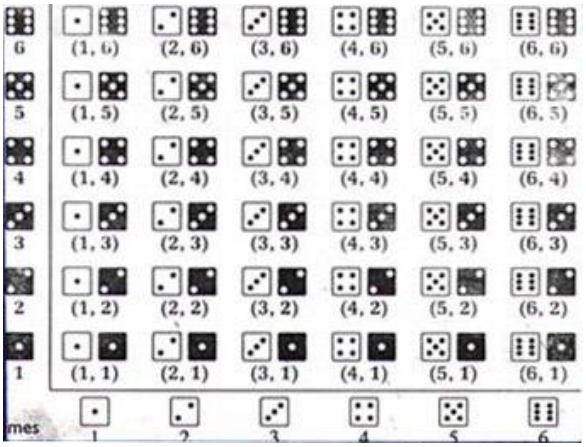
   D. P(Full-time | Part-time) = *Reduced Fraction:*

Determine whether each of the following are <u>Mutually Exclusive</u> or <u>Inclusive</u> and then determine the probability.

4. What is the probability of rolling a standard number cube to a number that is even or greater than 4?

Circle one of the following:	
Mutually Exclusive	Inclusive

Reduced Fraction:



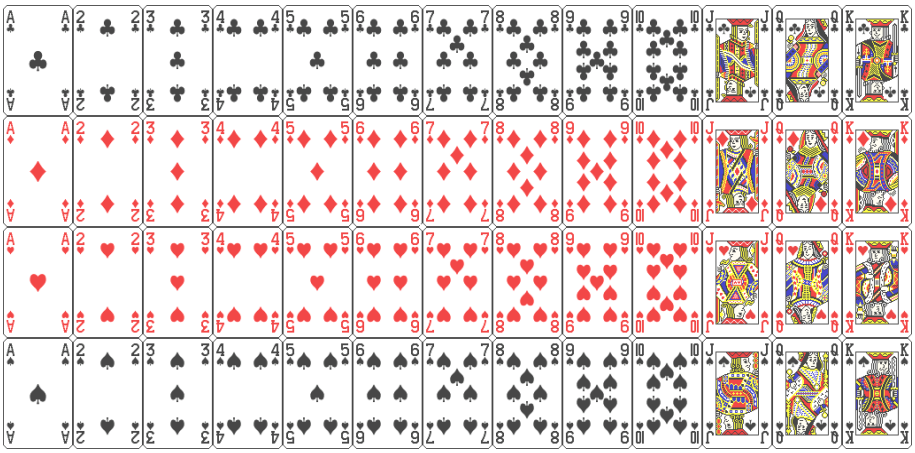
5. What is the probability of rolling two dice and having getting a sum of 4 OR getting a sum greater than 10?

6.

Circle one of the following:	
Mutually Exclusive	Inclusive

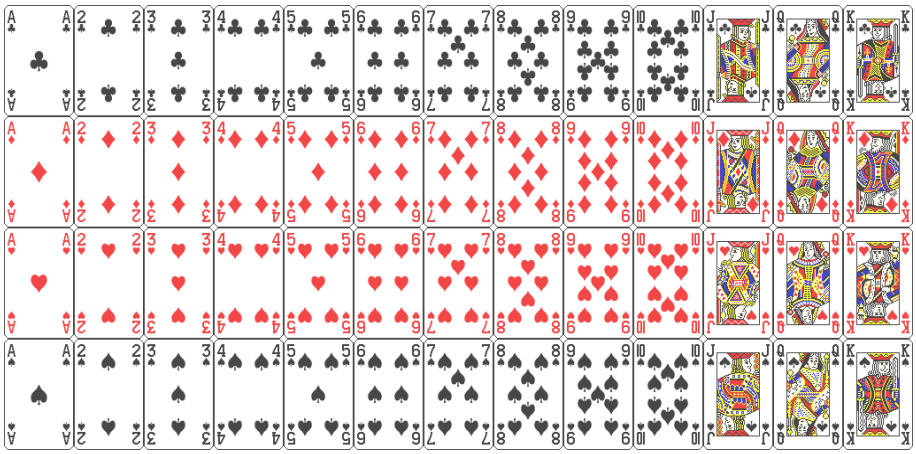
Reduced Fraction:

7. What is the probability of randomly selecting a card from a standard 52 card deck and having the card be a red card or a face card?



Circle one of the following:		*Reduced Fraction:*
Mutually Exclusive	Inclusive	

8. What is the probability of randomly selecting a card from a standard 52 card deck and having the card be a face card or an odd numbered card?

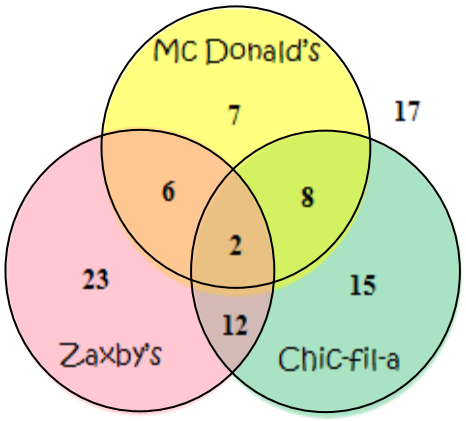


Circle one of the following:		*Reduced Fraction:*
Mutually Exclusive	Inclusive	

9. The following shows a VENN diagram with the results of a survey a teacher gave to all of her students. It represents where all of the students have gone to eat over the last month. What is the probability of the following?

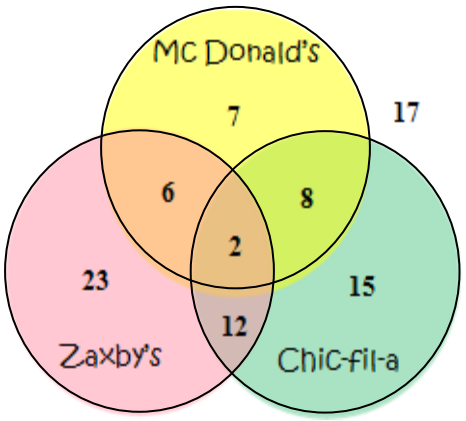
   i. What is the probability of randomly selecting a person from this group and picking a student that has NOT eaten at any of the restaurants OR they ate at McDonald's?

Circle one of the following:		*Reduced Fraction:*
Mutually Exclusive	Inclusive	



   ii. What is the probability of randomly selecting a person from this group and picking a student that has eaten at Mc Donald's OR Chick-fil-a?

Circle one of the following:		*Reduced Fraction:*
Mutually Exclusive	Inclusive	

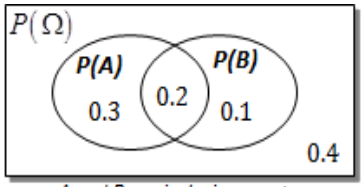


Sec 2.4 -Probability
VENN Diagrams Name:

MUTUALLY EXCLUSIVE (Disjoint) EVENTS vs. INCLUSIVE EVENTS

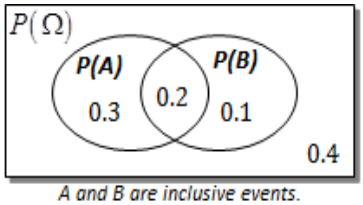
1. Consider the VENN diagrams at the right to help you answer the following.

A. P(A) = Decimal:



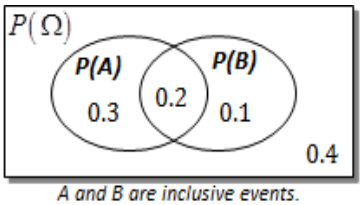
A and B are inclusive events.

B. P(A and B) = $P(A \cap B)$ = Decimal:



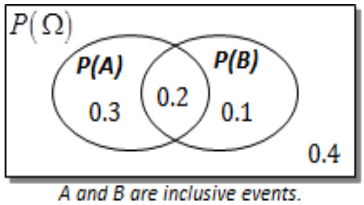
A and B are inclusive events.

C. P(A or B) = $P(A \cup B)$ = Decimal:



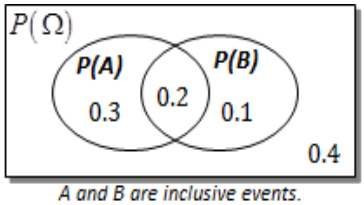
A and B are inclusive events.

D. P(A^c) = P(A') = Decimal:



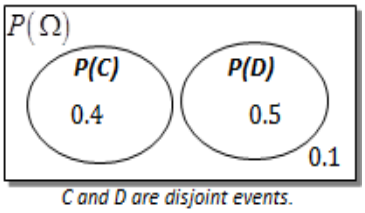
A and B are inclusive events.

E. P(A and B^c) = $P(A \cap B')$ = Decimal:



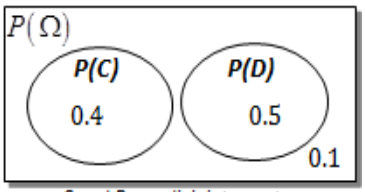
A and B are inclusive events.

F. P(C) = Decimal:



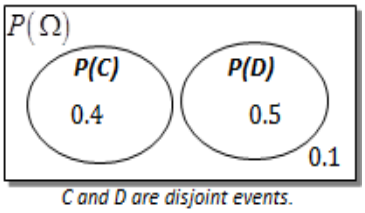
C and D are disjoint events.

G. P(C and D) = $P(C \cap D)$ = Decimal:



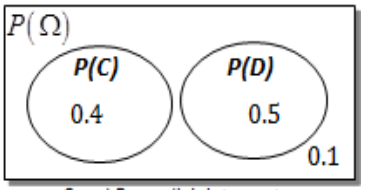
C and D are disjoint events.

H. P(C or D) = $P(C \cup D)$ = Decimal:



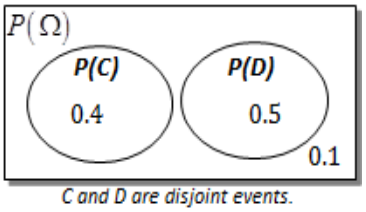
C and D are disjoint events.

I. P(C^c) = P(C') = Decimal:



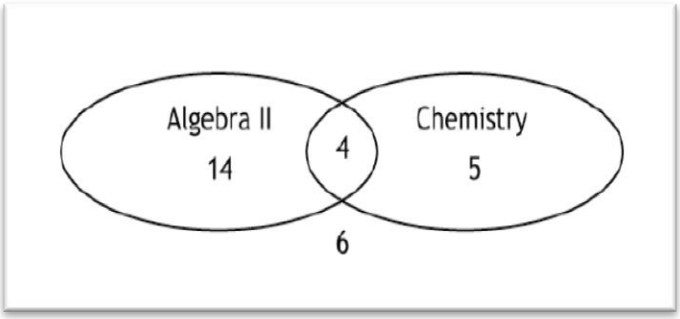
C and D are disjoint events.

J. P(C^c and D^c) = $P(C' \cap D')$ = Decimal:



C and D are disjoint events.

2. Ms. Snow conducted a survey of her homeroom. She asked students what math course and what science course they were taking this semester. Below are the results.



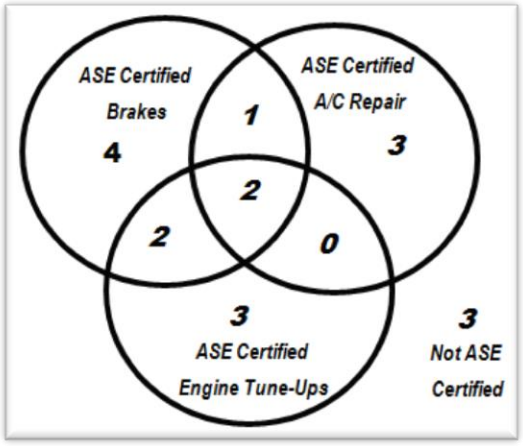
a. If a student is selected at random from Ms. Snow's homeroom, what is the probability that the student is taking Algebra II and Chemistry?

b. Find the probability : **P(Algebra II or Chemistry).**

c. If a student is selected at random from Ms. Snow's homeroom, what is the probability that the student is not taking either Algebra II or Chemistry? **P((Algebra II or Chemistry)^c)**

d. Find the probability of a student taking Chemistry, given that the student is not taking Algebra II, or **P(Chemistry | not taking Algebra II).**

3. A manager that owns 3 local area Car Maintenance Garages was researching certifications of mechanics that worked for her company. Consider the following Venn diagram.



a. What is the probability that a randomly selected mechanic from her 3 garages is ASE certified to work on Brakes? **P(Brakes)**

b. What is the probability that a randomly selected mechanic from her 3 garages is ASE certified to work on Brakes or A/C? **P(Brakes or A/C)**

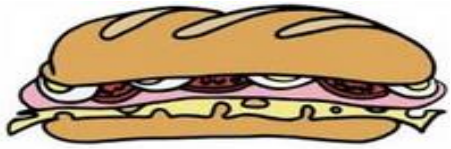
c. What is the probability that a randomly selected mechanic from her 3 garages is ASE certified to work on Brakes and Engine Tune-Ups? **P(Brakes and Tune-Ups)**

d. What is the probability that a randomly selected mechanic that is certified in Brakes given that the mechanic is certified to do Tune-Ups? **P(Brakes | Tune-Ups)**

Sec 2.5 -Probability
Tree Diagrams and Expected Value Name:

Tree Diagrams

Create-a-Sandwich Menu		
Bread	**Meat**	**Cheese**
White	Ham	American
Wheat	Turkey	Swiss
	Beef	Provolone
		Muenster



1. Create a tree diagram showing all possible sandwiches from the choices above if one of each category is used to create the sandwich.

2. What is the probability you will select a sandwich with white bread?

3. What is the probability that a customer will select a sandwich with white bread? Explain your reasoning.

4. If sandwiches are randomly made where each option is equally likely in each category, what is the probability you would get a sandwich with American cheese? Explain your reasoning.

5. If sandwiches are randomly made where each option is equally likely in each category, what is the probability you would get a sandwich on wheat bread with ham and any cheese? Explain your reasoning.

6. If sandwiches are randomly made where each option is equally likely in each category, what is the probability you would get a sandwich on white bread that has either beef or turkey and has Provolone cheese? Explain your reasoning.

7. If sandwiches are randomly made where each option is equally likely in each category, what is the probability you would get select a sandwich with neither beef nor Muenster cheese? Explain your reasoning.

At a car manufacturing plant each option is equally likely to be selected under each category to build a car.
Model: Toyonda Corvic

Trim Level	Exterior Color	Interior Color
DX	Black	Black
EX	Red	Gray
Si	Blue	
	White	

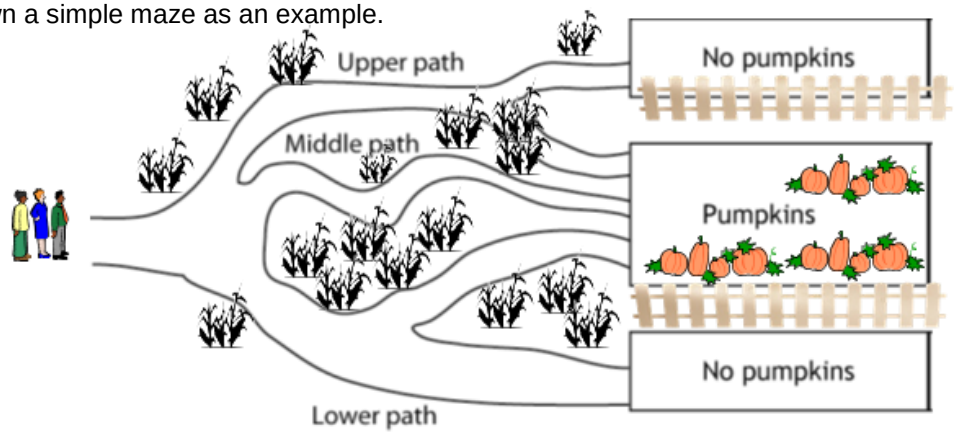
8. Create a tree diagram showing all possible versions of the car from the choices above if one of each category is used to create the car.

9. If each option under each category is equally likely to be selected, what is the probability that the first car being made on a given day has a Black Exterior and a Black Interior?

A church group in Washington state sells pumpkins every year to raise money for the children of their town. This year's crop, however, produced very small pumpkins. The group decided to construct a corn maze in a field and charge customers to walk through the maze.

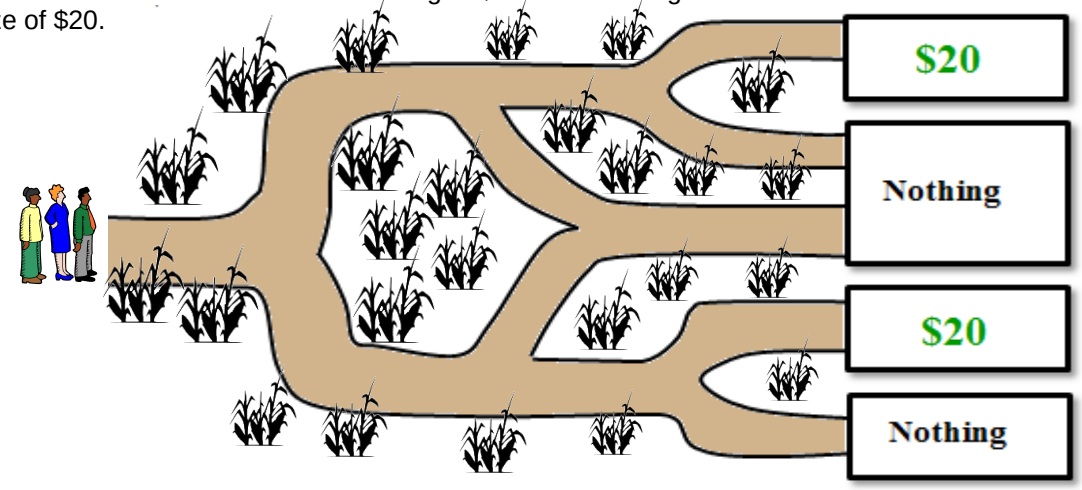
Customers can only walk forward. If the customers end up at an exit with pumpkins, they win a pumpkin. The church group asked some students to advise it on various possibilities of a customer getting a pumpkin.

Students were shown a simple maze as an example.



10. Make a tree diagram to show the group the possible paths customers might take, entering the maze on the upper, middle, or lower path and proceeding to an exit with or without a pumpkin (at each fork in the maze a person is equally likely to take any path. Finally, find the probability of ending at each gate.

11. A farmer created a corn maze. He charged $10 to run through the maze. Some of the trails led to winning a cash prize of $20.

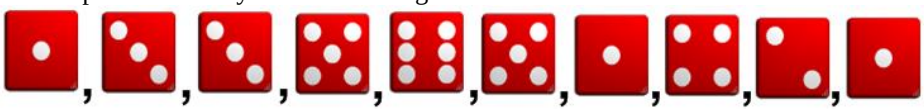


a. Make a tree diagram to show the group the possible paths customers might take, entering the maze on the upper, middle, or lower path and proceeding to an exit with or without a pumpkin (at each fork in the maze a person is equally likely to take any path. Finally, find the probability of ending at each gate.

b. Determine on average **expected** cost or gain to each person who runs through the maze.

Expected Values are weighted probability or percentages. How a teacher calculates your overall grade is done the same way an expected value.
Let's consider the expected value of a single standard 6-sided roll of a die.

EMPERICALLY: Roll a die several times and find the average value of all of the outcomes. Below is a sample of 10 rolls of a die. The more trials the closer the empirical should come to matching the theoretical expected value by the Law of Large Numbers.



Based on these trials the average result or approximate expected value would be:

$$E(A\ roll\ of\ a\ die) \approx \frac{1+3+3+5+6+5+1+4+2+1}{10} = \frac{31}{10} = 3.1$$

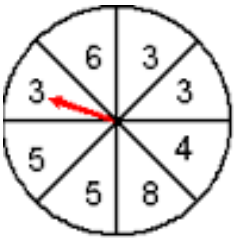
THEORETICALLY: We would find the probability of each event occurring and multiply it by the number occurring.

There is a $\frac{1}{6}$ chance of each number occurring.

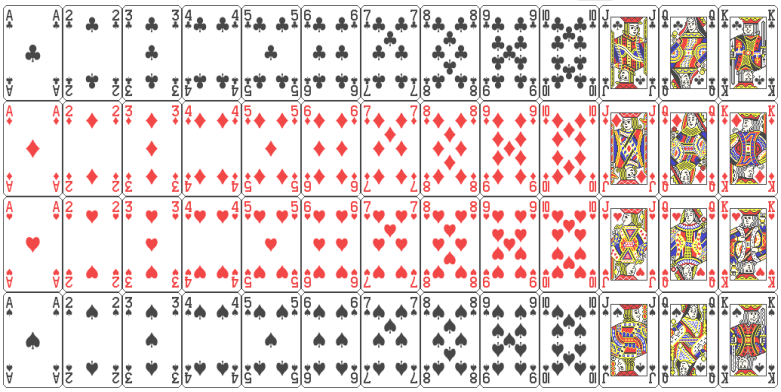
$$E(A\ roll\ of\ a\ die) = \left(\frac{1}{6}\right)\cdot 1 + \left(\frac{1}{6}\right)\cdot 2 + \left(\frac{1}{6}\right)\cdot 3 + \left(\frac{1}{6}\right)\cdot 4 + \left(\frac{1}{6}\right)\cdot 5 + \left(\frac{1}{6}\right)\cdot 6 = 3.5$$

12. Find the expected value of rolling a four sided tetrahedral die. Each of the four faces are equally likely to land on and the faces are numbered 1 through 4.

13. Find the expected value of the spinner shown. The spinner is equally likely to land on any of the 8 sectors.



14. In the card game 21, face cards count as 10 points, Aces count as 11 points, and number cards count the same number of points as the number on the card. Find the expected value of drawing the first card out of a shuffled deck of cards.



15. (GAME) A person created a game in which a player pays $2 to play. Then, they roll a standard six sided die. If it lands on a 5 or a 6, they get $5 (i.e. they get their original $2 back plus 3 more dollars). If they roll anything else, they just lose their $2 they paid to play. What is the expected value of each game to the player?

16. (GAME) A person created a game in which a player pays $4 to play. Then, they draw a card from a standard deck of shuffled cards. If they draw a numbered card they get $5 and if they draw an Ace they get $6. If they draw anything else they just lose the $4 they paid to play. What is the expected value of each game to the player?

17. (INSURANCE) An auto insurance agency charges males 16 – 18 year olds a base price of $1100 for insurance in a given year. Their accuracies have determined that there is a 8.5 % a claim will be made against such a policy at and an average claim of $4900. What does the insurance company expect to make on average off of each policy holder?

18. (BIDS) A construction company wants to submit a bid for remodeling a school. The research and planning needed to make the bid cost $4000. If the bid were accepted, the company would make a net profit of $26,000. Would you advise the company to spend the $4000 if the bid has only 20% probability of being accepted?

19. (INSURANCE) In many airports, a person can pay only $1.00 for a $100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays $100,000 if the insured person dies from a possible flight crash; otherwise the company gains $1.00 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers. How much can the company expect to gain on one policy? On 100,000 policies?

Sec 2.6 -Probability
Permutations, Combinations, and Binomial Probability

Name: _____

Counting Principle: *The counting principle suggests if one event has m possible outcomes and a second independent event has n possible outcomes, then there are m x n total possible outcomes for the two events together.*
(This basically suggests Options · Options · Options…… will determine the number of possible outcomes.)

Permutations: *A collection of things, in which the **order does matter**.*

Combinations: A collection of things, in which the **order does NOT matter**.

Determine first which to use Counting Principle, Permutations, or Combinations. Then, determine the number of possibilities:

1. 10 friends are at a party and they decided the party was boring and decided to go to a club in two different cars. One of the friends, Susan, has an SUV that can carry 7 passengers. So, Susan, can take 6 more of the 9 friends with her. How many different **groups** of 6 people are possible from the 9 remaining friends?

Circle one of the following:		
counting principle	Permutations	Combinations

2. A person is playing a card game called '21' in which they receive 2 cards from a dealer. If the **order doesn't matter** how many 2 card hands are possible (52 cards in a deck)?

Circle one of the following:		
counting principle	Permutations	Combinations

3. There are 12 students are in a class. The students are instructed to line up on a set chorus stand. That has 3 rows. The front row can hold 5 people. How many possible **arrangements** are there for the front row using the 12 students?

Circle one of the following:		
counting principle	Permutations	Combinations

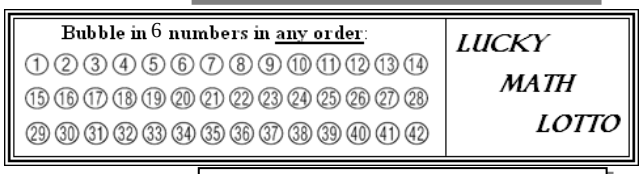
4. How many unique license plates tags can be created if each is 6 characters such that the first three characters can be any letter or number but no character is allowed to be repeated in the first three characters, and the last three spaces can be any letter or number?

Circle one of the following:		
counting principle	Permutations	Combinations

5. The Mrs. Georgia pageant is taking place and there are 20 contestants. How many different ways could the pageant have a winner and a runner up?

Circle one of the following:		
counting principle	Permutations	Combinations

6. The following shows a lottery card in which a participant bubbles in 6 numbers out of 42. How many different ways can a participant select a **group** of 6 numbers?



Circle one of the following:		
counting principle	Permutations	Combinations

7. A Chocolate Ice Cream Sunday is being prepared at Bruce & Stirs. At the ice cream shop there are 10 different types of ice cream that can be used, 3 different types of syrup, and 2 different types of nuts. How many different types of ice cream Sundays are possible if each contain **one** type of ice cream, one type of syrup, and one type of nut?

Circle one of the following:		
counting principle	Permutations	Combinations

8. A Chocolate Ice Cream Sunday is being prepared at Bruce & Stirs. At the ice cream shop there are 10 different types of ice cream that can be used, 3 different types of syrup, and 2 different types of nuts. How many different types of ice cream Sundays are possible if each contain **two** types of ice cream, one type of syrup, and one type of nut?

Circle one of the following:		
counting principle	Permutations	Combinations

9. **Binomial Probabilities** A student is taking a 4 question *true/false* quiz. The student doesn't know any of the answers and completely guesses on each question.
   a. Create a tree diagram with probabilities showing every possible outcome of Correct and Wrong answers.

   b. What is the probability that the student will make EXACTLY a 75% using your **tree diagram**?

   c. What is the probability that the student will make EXACTLY a 75% using your **combinations or the calculator**?

10. **Binomial Probabilities** A student is taking a 4 question *multiple choice* quiz with 4 choices each. The student doesn't know any of the answers and completely guesses on each question.
   a. Create a tree diagram with probabilities showing every possible outcome of Correct and Wrong answers.

   b. What is the probability that the student will make EXACTLY a 50% using your **tree diagram**?

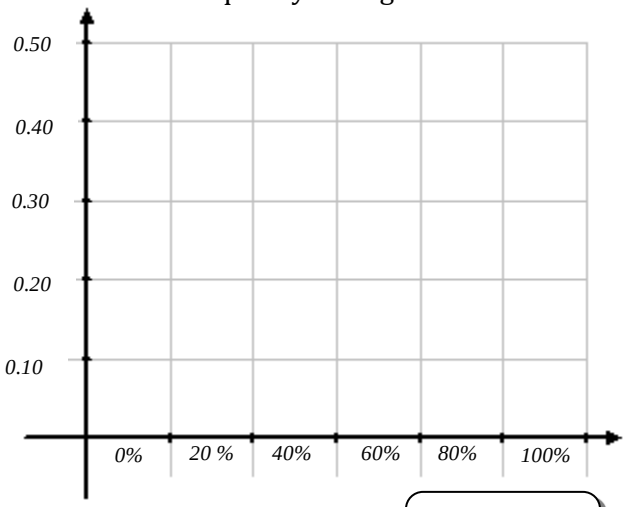
   c. What is the probability that the student will make EXACTLY a 0% using your **combinations or the calculator**?

11. A student is taking a 5 question *multiple choice* quiz (with 5 possible answers each). If the student COMPLETELY guesses on each question on the quiz fill out the following discrete probability distribution table. (PDF)

A.

Grade	Probability
0 %	
20%	
40%	
60%	
80%	
100%	

B. Make a Relative Frequency Histogram.



C. What is probability that the student will make a failing score (60% or below)? (CDF)

12. A surgical technique is performed on **seven** patients. You are told there is a 70% chance of success. Find the probability that the surgery is successful for:

a. Exactly five patients? $P(x = 5)$ =

b. Less than five patients? $P(x \leq 4)$ =

c. At least three patients? $P(x \geq 3)$ =

13. A manufacture sells a can of fix-a-flat to repair flat tires. Using fix-a-flat to repair a tire works 78% of the time. A person driving an 18 wheeler ran over a bunch of nails. All of the tires went flat. However, luckily the driver has 18 cans of fix-a-flat.

a. What is the probability that all 18 tires are successfully repaired? (PDF)

b. What is the probability that exactly 10 tires are inflated (enough to drive)? (PDF)

c. What is the probability that at least 12 tires are inflated? (CDF)

14. Jeff creates a game in which the player rolls 5 dice.

a. What is the probability in this game of having exactly 2 dice land on six?

b. What is the probability in this game of having 3 or more dice land on six?

15. Mark creates a game in which 4 pennies are flipped at the same time.

a. 3 points is awarded if all 4 pennies lands heads up. What is the probability of all 4 pennies landing heads up?

b. 2 points is awarded if 3 pennies lands heads up. What is the probability of exactly 3 pennies landing heads up?

c. No points are awarded for anything else. What is the probability of getting no points?

d. Find the expected value of points for each play of the game.

Sec 2.7 -Probability
Zero-Sum Games

Name:

A zero-sum game consists of a game in which the total value or amount of the assets to be divided up between players never changes.

A group of people playing a typical card game with wagers is a good example of a zero-sum game. Let's consider 3 players (Ann, Bob, Chris) sitting at the table playing cards such that each player starts with $20 for a total of $60 at the table.

Ann		Bob		Chris		POT		TOTAL
$20	+	$20	+	$20	+	$0	=	$60
-3		-3		-3		+3 +3 +3		

On the first round of play, each player has contributed $3 to the pot (the amount of money to be won for the round) for a total of $9 in the pot and each player still has $17 left and the total is still $3 \cdot \$17 + \$9 = \$60$

Ann		Bob		Chris		POT		TOTAL
$17	+	$17	+	$17	+	$9	=	$60
+9						-9		

Ann wins the first round and collects the $9 in the pot. So, she would now have $26 and everyone else would have $17 and the total amount at the table is still $60. When somebody wins an amount other members must take a loss in the exact same amount (so the net change is a zero-sum).

Ann		Bob		Chris		POT		TOTAL
$26	+	$17	+	$17	+	$0	=	$60

In every step of the game above, the total value of assets for the game always totaled $60. The assets may have been redistributed but the total remained the same.

Let's create another zero-sum game. In this game two participants (Alex and Bianca) each have 4 quarters. For each turn of the game each of the players holds out one of their quarters at the same time. If the quarters are the same side up, Alex wins and keeps both quarters. If the quarters are opposite sides up, Bianca wins and keeps both quarters.

The chart or pay off matrix shows the potential outcomes:

Commonly, the payoff matrix consists of the options or strategies that can be applied to the game to be listed in the row and column heading titles.

The elements of the matrix usually consist of the payout for person in the row (e.g. Alex) heading first followed by the payout for the person in the column (e.g. Bianca) heading.

Sometimes, just one number is listed in each element of the matrix for a zero-sum game since we know the 2 values will just differ by a sign. If only 1 number is used for each element of the matrix, the pay offs are listed for the person in the row (e.g. Alex) heading.

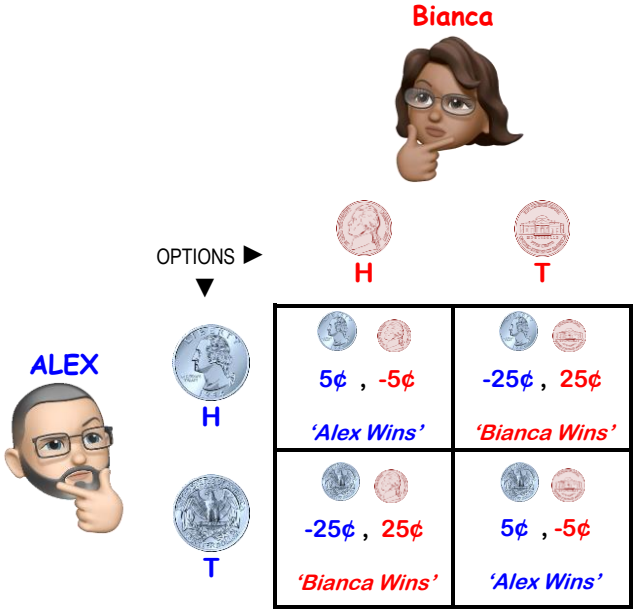
Pay Off Matrix

OPTIONS ▶ ▼

25¢ , -25¢ 'Alex Wins'	-25¢ , 25¢ 'Bianca Wins'
-25¢ , 25¢ 'Bianca Wins'	25¢ , -25¢ 'Alex Wins'

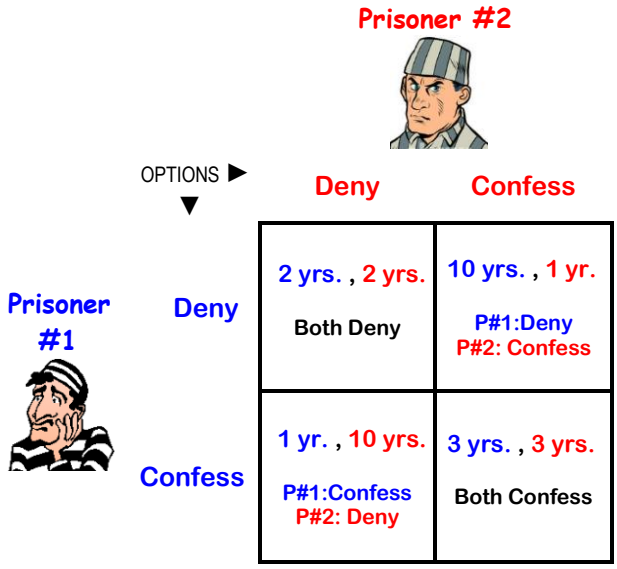
While it might be a little unfair, we could have Bianca use a nickel while Alex still uses a quarter, and it would continue to be a zero-sum game. Again, if both coins are the heads up or tails up, Alex collects both coins but if both coins are opposite side up, Bianca collects both coins. The payoff matrix might look a little different which is shown at the right.

Create a Payoff matrix, using only one number per matrix element for a similar game but where Alex uses a penny and Bianca uses a dime.



A common example of a non-zero-sum game is the prisoner's dilemma. Two people are caught red handed together stealing $4000 from a store. Both people are immediately sentenced to 2 years in prison. A detective recognizes both prisoners as the potential suspects in another case that involved a much larger bank robbery. The detective offers to cut a deal with each prisoner separately. The detective tells each one if they confess confidentially that the other prisoner was involved in the bank robbery, that their sentence could potentially be reduced to 1 year but only if they confess and their partner does not. The partner that did not confess in this scenario would be sentenced to 10 years in prison. If both confess or testify against each other, they will each be sentenced to a total of 3 years.

In this example, the payoffs for each outcome don't sum to zero and is therefore not a zero-sum game.



- Which outcome would have the best overall net outcome for the 2 prisoners? (Explain your answer.)

- If you were a consultant to prisoner #1, what would you recommend that he do without knowledge of what prisoner #2 might do? (Explain your answer.)

- What is the Nash Equilibrium Outcome for this game?