Sec 7.1 – VECTORS

Basic Vector Forms

Name:

1. Describe each of the following vectors in rectangular form $\langle x, y \rangle$ and using component vectors *i* and *j* (e.g. 3*i* + 4*j*)

A. Describe Vector $\overrightarrow{AB}$: A:(-6, 2) B:(-1,5)

$\overrightarrow{AB}$: ⟨-1--6, 5-2⟩: ⟨5, 3⟩ : $5i + 3j$

B. Describe Vector $\overrightarrow{CD}$: C:(5,6) D:(1,4)

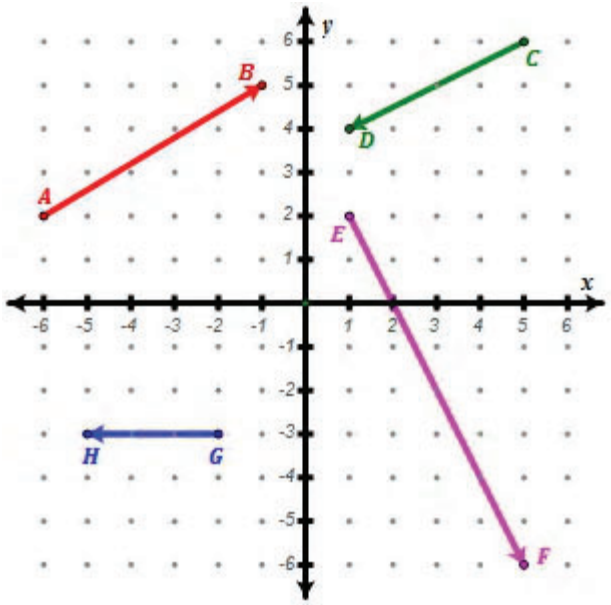
$\overrightarrow{CD}$: ⟨1-5, 4-6⟩: ⟨-4, -2⟩ : $-4i - 2j$

C. Describe Vector $\overrightarrow{EF}$: E:(1, 2) F: (5,-6)

$\overrightarrow{EF}$: ⟨5-1, -6-2⟩: ⟨4, -8⟩ : $4i - 8j$

D. Describe Vector $\overrightarrow{GH}$: G:(-2,-3) H: (-5, -3)

$\overrightarrow{GH}$: ⟨-5--2, -3--3⟩: ⟨-3, 0⟩ : $-3i$



2. Describe each of the following vectors in rectangular form $\langle x, y \rangle$ and using component vectors *i* and *j* (e.g. 3*i* + 4*j*)

A. Describe Vector $\overrightarrow{MN}$ given point M(– 4, 2) and N(2, 6)

$\overrightarrow{MN}$: ⟨2--4, 6-2⟩

⟨6 , 4⟩

$6i + 4j$

B. Describe Vector $\overrightarrow{ST}$ given point S(3, – 2) and T (– 4, 6)

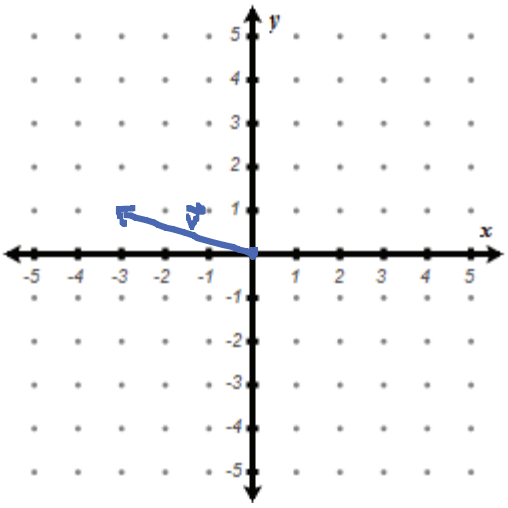
$\overrightarrow{ST}$: ⟨-4-3, 6--2⟩

⟨-7, 8⟩

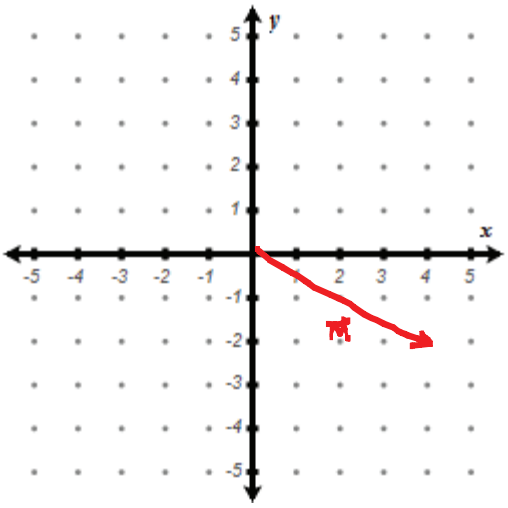
$-7i + 8j$

3. Create a graph of each of the following vectors in standard form.

A. The vector $\vec{v}$: ⟨−3,1⟩



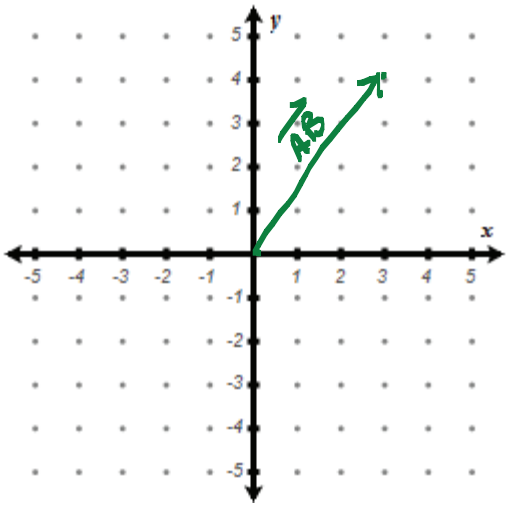
B. The vector $\vec{m}$: $4\boldsymbol{i} - 2\boldsymbol{j}$



C. The vector $\overrightarrow{AB}$ given point A(–2, 2) and B(1, 6)

$\overrightarrow{AB}$: ⟨1--2, 6-2⟩

: ⟨3, 4⟩

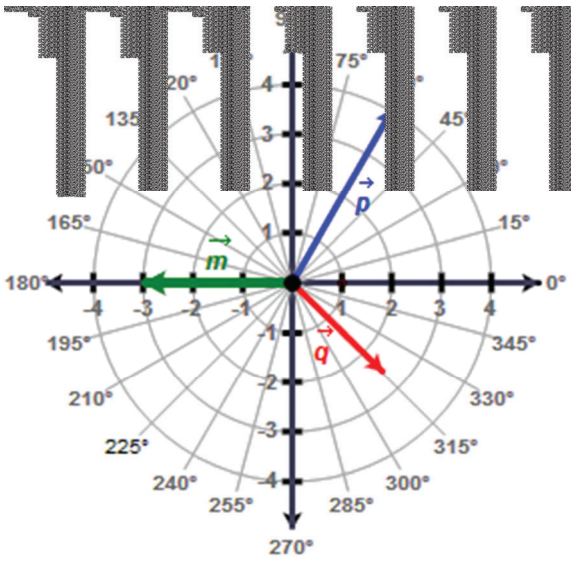


4. Describe each of the following vectors in <u>polar form</u> $\langle r, \vartheta \rangle$.

A. Describe Vector $\vec{p}$: $\langle 4, 60° \rangle$

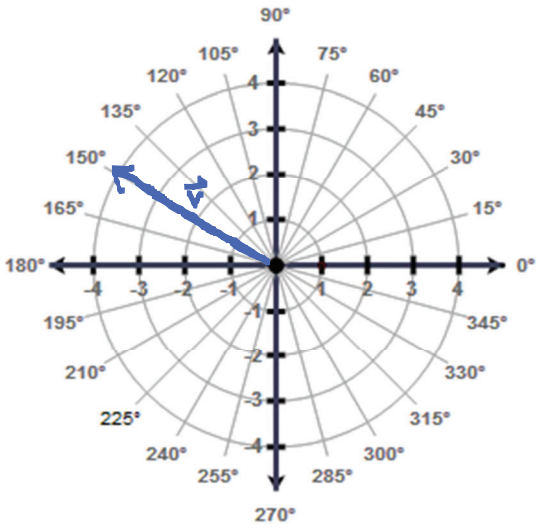
B. Describe Vector $\vec{q}$: $\langle 2.5, 315° \rangle$

C. Describe Vector $\vec{m}$: $\langle 3, 180° \rangle$ or $\langle -3, 0° \rangle$

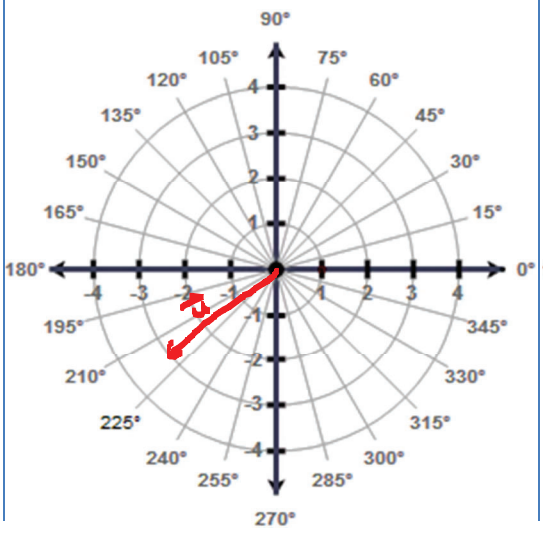


5. Create a graph of each of the following vectors in standard form.

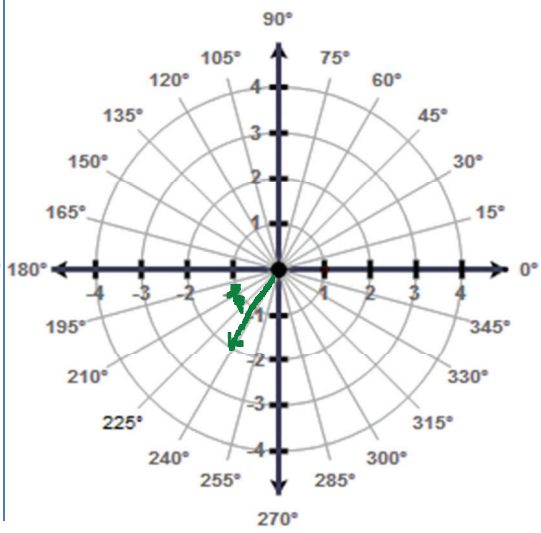
A. The vector $\vec{v}: \langle 4, 150° \rangle$



B. The vector $\vec{u}: \langle 3, -140° \rangle$



C. The vector $\vec{p}: \langle -2, 60° \rangle$



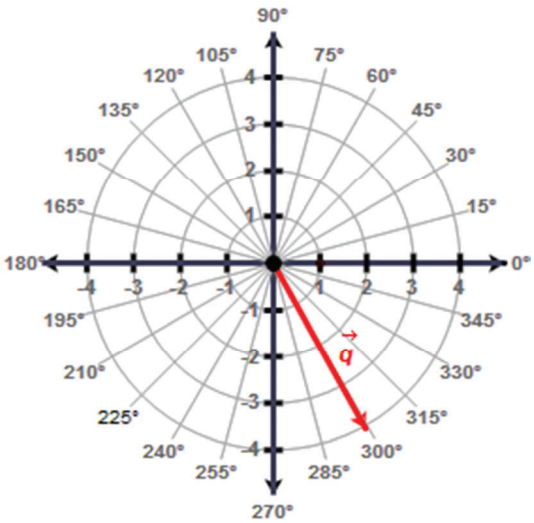
6. Determine at least 4 ways to describe vector $\vec{q}$ in polar form:

$\vec{q}: \langle 4, 300° \rangle$

$\vec{q}: \langle 4, -60° \rangle$

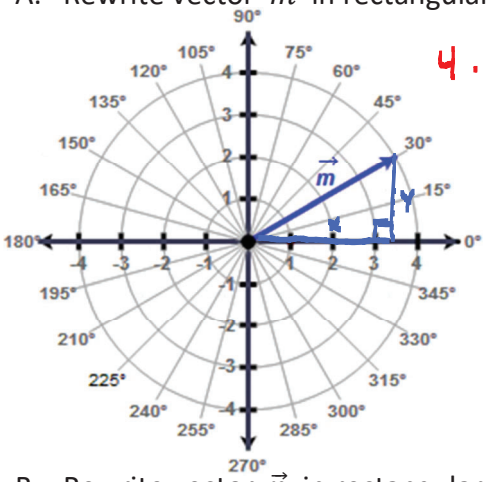
$\vec{q}: \langle -4, 120° \rangle$

$\vec{q}: \langle -4, -240° \rangle$



7. Rewrite each of the following vectors from polar form to rectangular form.

A. Rewrite vector $\vec{m}$ in rectangular form and graph it on the rectangular graph paper in standard position.



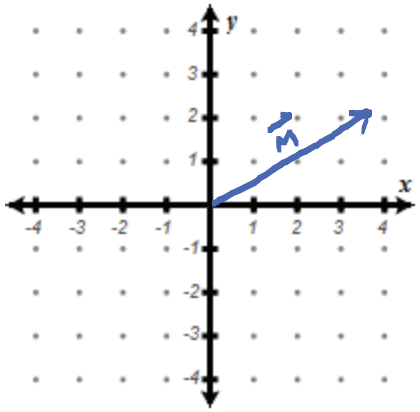
$4 \cdot \cos 30 = \frac{x}{4} \cdot 4$ $\quad 4 \cdot \sin 30 = \frac{y}{4} \cdot 4$

$\vec{m}: \langle 4\cos 30°, 4 \sin 30° \rangle$

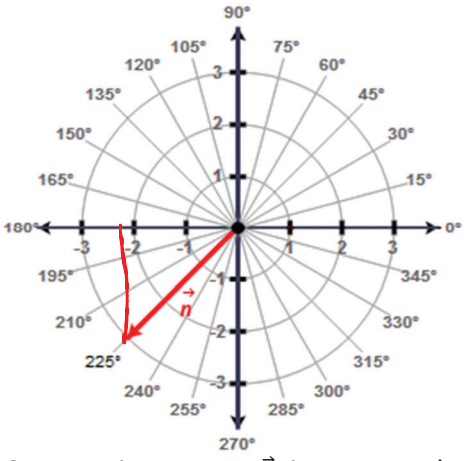
$\vec{m}: \langle 4 \cdot \frac{\sqrt{3}}{2}, 4 \cdot \frac{1}{2} \rangle$

$\vec{m}: \langle 2\sqrt{3}, 2 \rangle$

$\vec{m}: \langle 3.46, 2 \rangle$



B. Rewrite vector $\vec{n}$ in rectangular form and graph it on the rectangular graph paper in standard position.

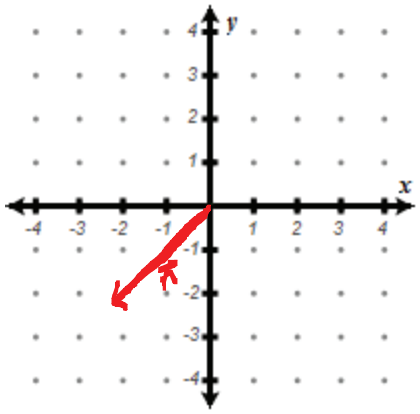


$\vec{n}: \langle 3\cos 225°, 3 \sin 225° \rangle$

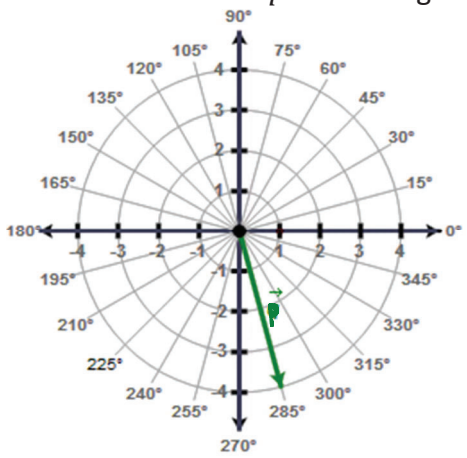
$\vec{n}: \langle 3 \cdot \frac{-\sqrt{2}}{2}, 3 \cdot \frac{-\sqrt{2}}{2} \rangle$

$\vec{n}: \langle \frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2} \rangle$

$\vec{n}: \langle -2.12, -2.12 \rangle$



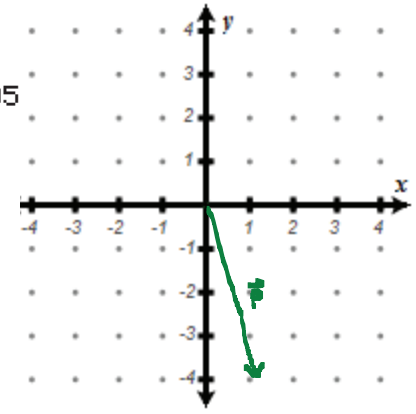
C. Rewrite vector $\vec{p}$ in rectangular form and graph it on the rectangular graph paper in standard position.



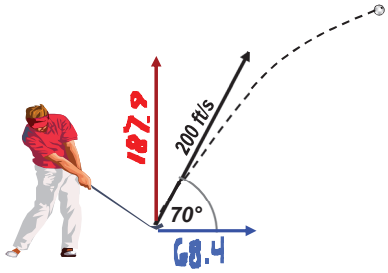
$\vec{p}: \langle 4\cos 285°, 4 \sin 285° \rangle$

4cos(285) 1.03527618 $\quad$ 4sin(285) -3.863703305

$\vec{p}: \langle 1.04, -3.86 \rangle$



8. Consider the golfer below. He struck the ball so that it was moving at a speed of 200 feet per second at a 70° angle.



A. Write the vector in polar form. $\langle 200, 70° \rangle$

B. Write the vector in rectangular form. $\langle 200\cos 70°, 200 \sin 70° \rangle$

$\langle 68.4, 187.9 \rangle$

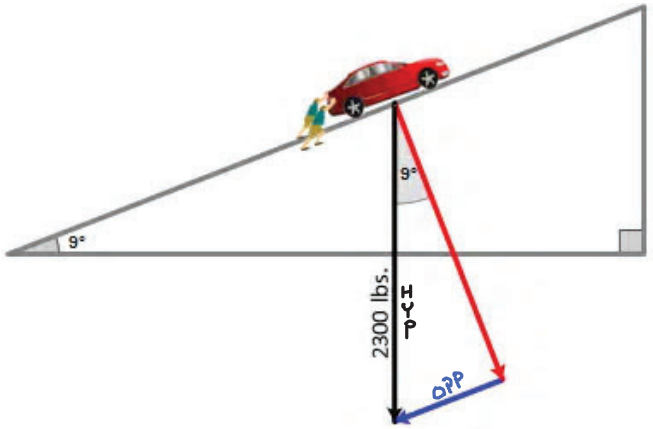
C. If we assumed that the green was 800 feet away horizontally, how long would it take the golf ball to reach the green (assuming air resistance wasn't a factor)?

$d = r\,t$

$\frac{800}{68.4} = \frac{68.4\,t}{68.4}$

$11.7 \text{ sec} = t$

9. Consider the ramp shown below. A car that weighs 2300 pounds is being pushed up a ramp with a 9° elevation.



How many pounds of force must the people use just to hold the car in place on the ramp?

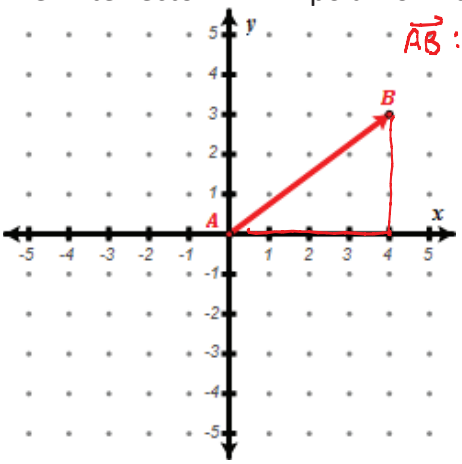
$\sin(9) = \frac{x}{2300}$

$x = 2300 \cdot \sin(9)$

$x \approx 359.8$ POUNDS

10. Rewrite each of the following vectors from rectangular form to polar form.

A. Rewrite vector $\overrightarrow{AB}$ in polar form and graph it on the polar graph paper in standard position.



$\overrightarrow{AB}: \langle 4, 3 \rangle$

$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

$\sqrt{25} = \sqrt{c^2}$

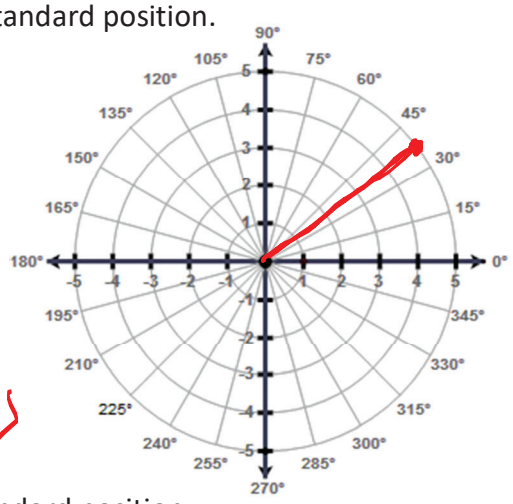
$5 = c$

$\tan\alpha = \frac{3}{4}$

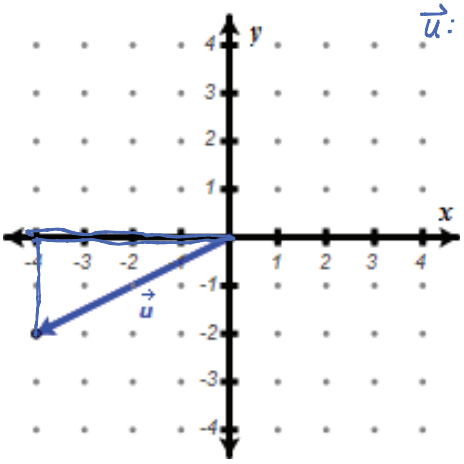
$\alpha = \tan^{-1}(\frac{3}{4})$

$\alpha = 36.9°$

$\langle 5, 36.9° \rangle$



B. Rewrite vector $\vec{u}$ in polar form and graph it on the polar graph paper in standard position.



$\vec{u}: \langle -4, -2 \rangle$

$\tan(\alpha) = \frac{|-2|}{|-4|}$

$\alpha = \tan^{-1}(\frac{2}{4})$

$\alpha \approx 26.57°$

$\theta \approx 180 + 26.57$

$\theta \approx 206.57$

$4^2 + 2^2 = c^2$

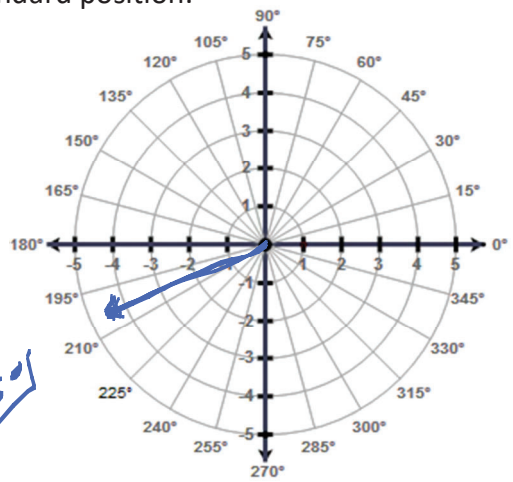
$16 + 4 = c^2$

$20 = c^2$

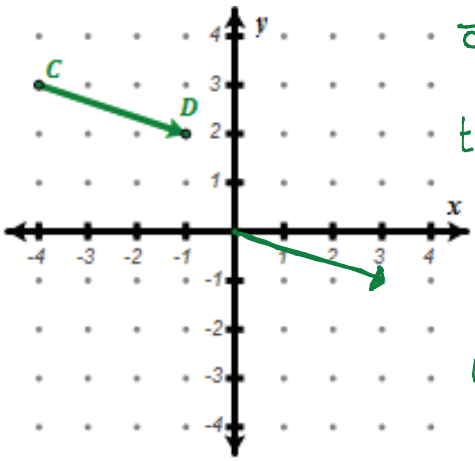
$\sqrt{20} = c$

$4.47 \approx c$

$\langle 4.47, 206.6° \rangle$



C. Rewrite vector $\overrightarrow{CD}$ in polar form and graph it on the polar graph paper in standard position.



$\overrightarrow{CD}: \langle 3, -1 \rangle$

$\tan(\alpha) = \frac{|-1|}{|3|}$

$\alpha = \tan^{-1}(\frac{1}{3})$

$\alpha \approx 18.43°$

$\theta = 360 - 18.43°$

$\theta \approx 341.57°$

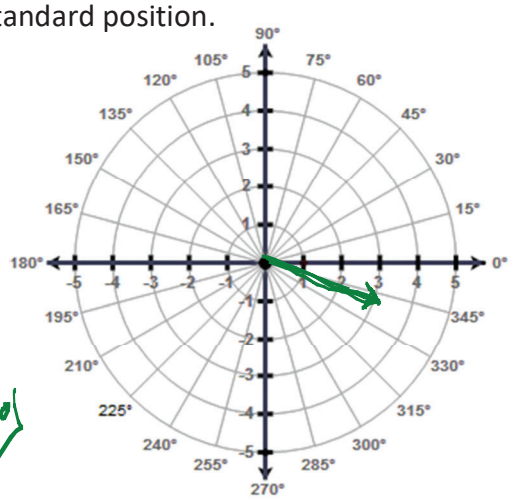
$3^2 + 1^2 = c^2$

$9 + 1 = c^2$

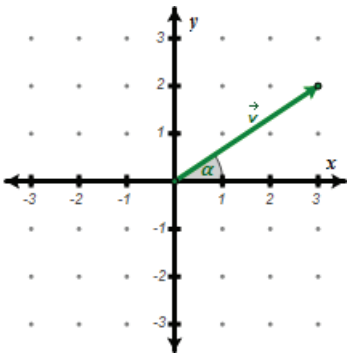
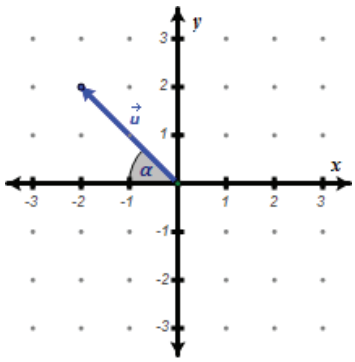
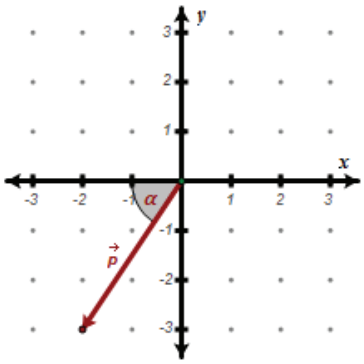
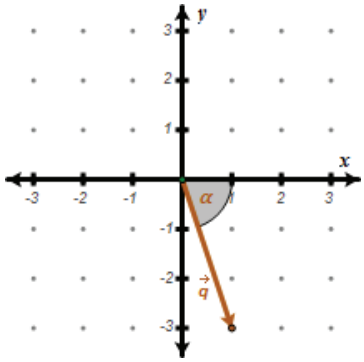
$\sqrt{10} = \sqrt{c^2}$

$3.16 \approx c$

$\langle 3.16, 341.57° \rangle$



Finding Theta *(where α is the reference angle)*

Quad 1 (x & y positive) *e.g.* $\vec{v}:\langle 3,2\rangle$	Quad 2 (x negative, y positive) *e.g.* $\vec{u}:\langle -2,2\rangle$	Quad 3 (x & y negative) *e.g.* $\vec{p}:\langle -2,-3\rangle$	Quad 4 (x negative, y positive) *e.g.* $\vec{q}:\langle 1,-3\rangle$
			
$\tan^{-1}\left(\frac{\lvert 2\rvert}{\lvert 3\rvert}\right)=\alpha\approx 33.7°$	$\tan^{-1}\left(\frac{\lvert 2\rvert}{\lvert -2\rvert}\right)=\alpha=45°$	$\tan^{-1}\left(\frac{\lvert -3\rvert}{\lvert -2\rvert}\right)=\alpha\approx 56.3°$	$\tan^{-1}\left(\frac{\lvert -3\rvert}{\lvert 1\rvert}\right)=\alpha\approx 71.6°$
$\theta\approx 33.7°$	$\theta=180°-45°=135°$	$\theta=180°+56.3°$ $\approx 236.3°$	$\theta=360°-71.6°\approx 288.4$
$\theta=\alpha$	$\theta=180°-\alpha$	$\theta=180°+\alpha$	$\theta=360°-\alpha$

11. Perform the following vector operations.

A. Consider $\vec{m}:\langle -7,4\rangle$ determine $\|\vec{m}\|$.

$(-7)^2+(4)^2=c^2$

$49+16=c^2$

$\sqrt{65}=\sqrt{c^2}$

$\|\vec{m}\|=\sqrt{65}$

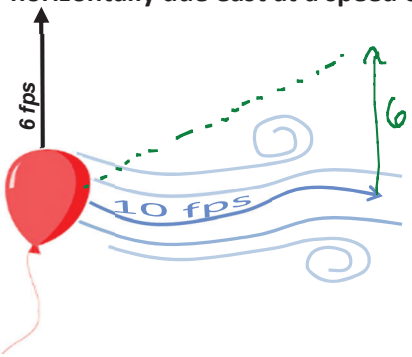
≈ 8.06

B. Rewrite vector $\vec{n}:\langle 9, 260°\rangle$ in rectangular form.

$\vec{n}:\langle 9\cos 260,\ 9\sin 260\rangle$

$:\langle -1.56,\ -8.86\rangle$

12. A balloon is floating up in to the sky at a rate of 6 feet per second. At the same time the wind is blowing the balloon horizontally due east at a speed of 10 feet per second.



A. How fast is the balloon actually moving?

$(6)^2+(10)^2=c^2$

$36+100=c^2$

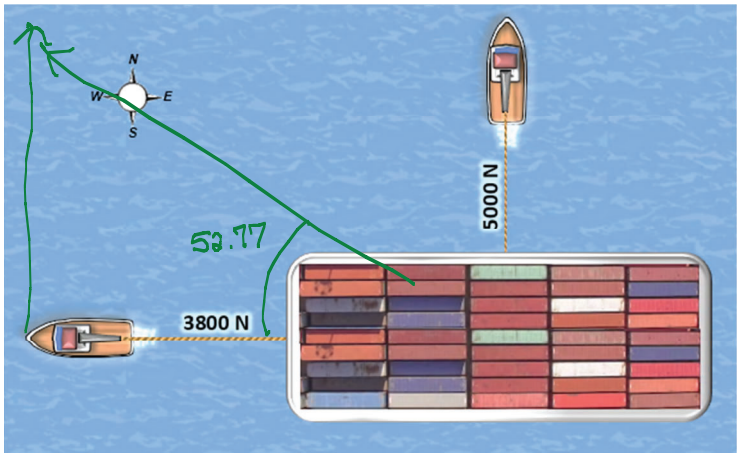
$\sqrt{136}=\sqrt{c^2}$

$c\approx 11.66$ fps

B. What is the angle of elevation of the balloon's ascent?

$\tan(\alpha)=\frac{|6|}{|10|}$, $\alpha=\tan^{-1}\left(\frac{6}{10}\right)\approx 30.96°$

13. A barge full of containers is being moved across a bay. One tow boat is pulling the barge due North with a force of 5000 Newtons. A second tow boat is pulling the barge due West with a force of 3800 Newtons.



A. In what direction will the barge move?

$\tan(\alpha)=\frac{|5000|}{|3800|}$

$\alpha\approx 52.77°$

$\theta=180-52.77$

$\theta\approx 127.23°$

OR $52.77°$ NORTH OF WEST.

B. How much force is being applied in that directions?

$(3800)^2+(5000)^2=c^2$

$\sqrt{39440000}=\sqrt{c^2}$

$6280.1\text{ N}\approx c$