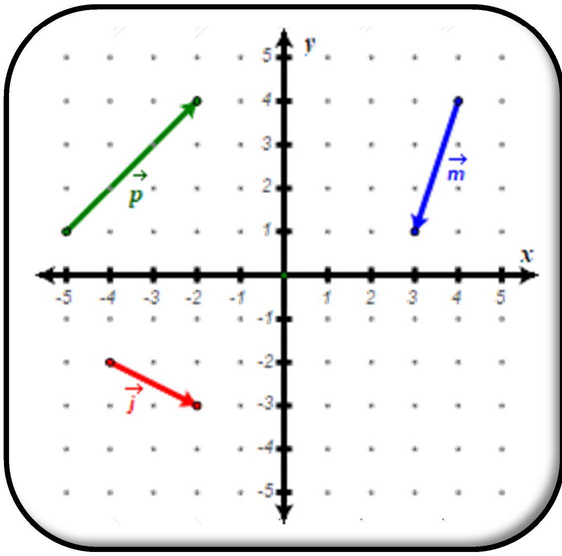
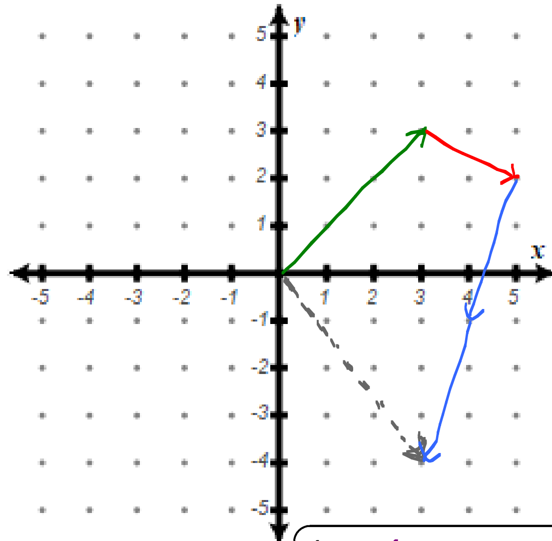
1. Create the following vector statement in the graph to determine the resultant vector in rectangular form.

A. Using the information below graphically determine $\vec{p} + \vec{j} + 2\vec{m}$



Show work here



1a. $\langle 3, -4 \rangle$

These should be the same

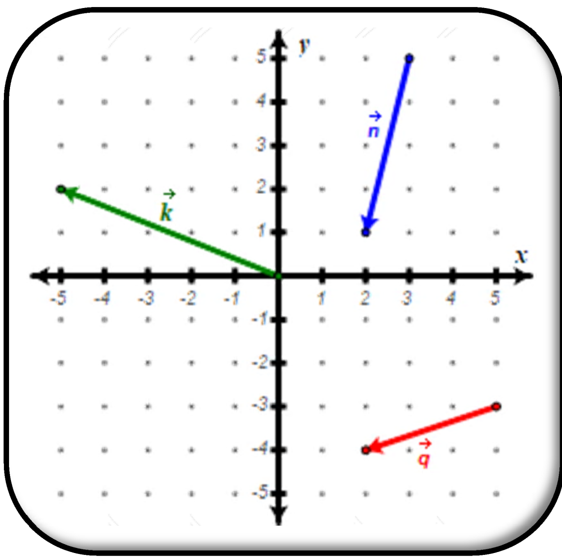
B. Algebraically find $\vec{p} + \vec{j} + 2\vec{m}$

$\vec{m} : \langle -1, -3 \rangle$

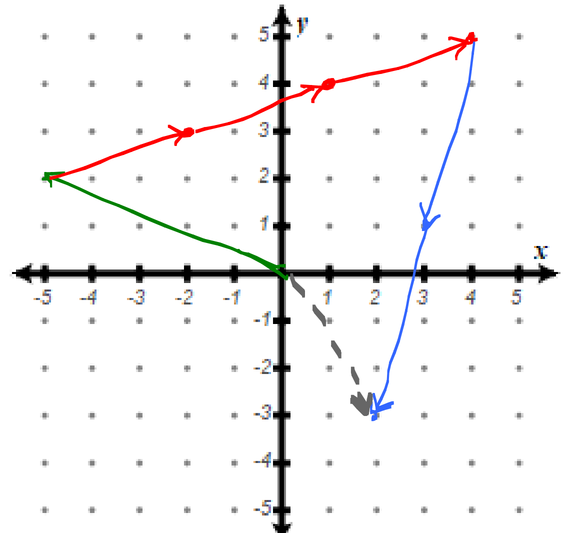
$$
\begin{array}{rl}
\vec{p} : & \langle 3, 3 \rangle \\
\vec{j} : & \langle 2, -1 \rangle \\
+ \quad 2\vec{m} : & \langle -2, -6 \rangle \\
\hline
 & \langle 3, -4 \rangle
\end{array}
$$

1b. $\langle 3, -4 \rangle$

C. Using the information below graphically determine $\vec{k} - 3\vec{q} + 2\vec{n}$



Show work here



1c. $\langle 2, -3 \rangle$

These should be the same

D. Algebraically find $\vec{k} - 3\vec{q} + 2\vec{n}$

$\vec{q} : \langle -3, -1 \rangle$

$\vec{n} : \langle -1, -4 \rangle$

$$
\begin{array}{rl}
\vec{k} : & \langle -5, 2 \rangle \\
-3\vec{q} : & \langle 9, 3 \rangle \\
+ \quad 2\vec{n} : & \langle -2, -8 \rangle \\
\hline
 & \langle 2, -3 \rangle
\end{array}
$$

1d. $\langle 2, -3 \rangle$

2. Let the following vectors be defined: $\vec{v}: \langle -3, 5\rangle$ $\vec{u}: \langle 7, 2\rangle$ $\vec{p}: \langle 4, -9\rangle$

A. Simplify the following vector expression $\vec{v} + 2\vec{u}$ and write your answer in component form.

$\vec{v}: \langle -3, 5\rangle$

$2\vec{u}: \langle 14, 4\rangle$

2a. $\langle 11, 9\rangle$

B. Simplify the following vector expression $\|\vec{p} - \vec{v}\|$ (i.e. What is the magnitude of the resultant vector?)

$\vec{p}: \langle 4, -9\rangle$

$-\vec{v}: \langle 3, -5\rangle$

$\langle 7, -14\rangle$

$c^2 = (7)^2 + (-14)^2$

$\sqrt{c^2} = \sqrt{245}$

$c = 7\sqrt{5}$

2b. $c \approx 15.65$

C. Find the direction of the vector expression $\vec{v} + \vec{u} + \vec{p}$.

$\vec{v}: \langle -3, 5\rangle$

$\vec{u}: \langle 7, 2\rangle$

$+\ \vec{p}: \langle 4, -9\rangle$

$\langle 8, -2\rangle$

2c. $\langle 8, -2\rangle$

D. Simplify the following vector expression $2\vec{u} + \vec{p}$ and write your answer in polar form.

$2\vec{u}: \langle 14, 4\rangle$

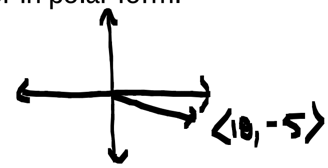
$+\ \vec{p}: \langle 4, -9\rangle$

$\langle 18, -5\rangle$

$(18)^2 + (-5)^2 = c^2$

$\sqrt{349} = \sqrt{c^2}$

$18.68 \approx c$



$\tan^{-1}\left(\frac{|-5|}{|18|}\right) \approx 15.5°$

$\theta \approx 360° - 15.5°$

$\theta \approx 344.5°$

2d. $\langle 18.7, 344.5°\rangle$

3. Let the following vectors be defined: $\vec{p}: \langle 4, 30°\rangle$ $\vec{q}: \langle 3, 120°\rangle$

A. Rewrite vector $\vec{p}$ in rectangular form.

$\langle 4\cos 30°, 4\sin 30°\rangle$

3a. $\langle 3.46, 2\rangle$

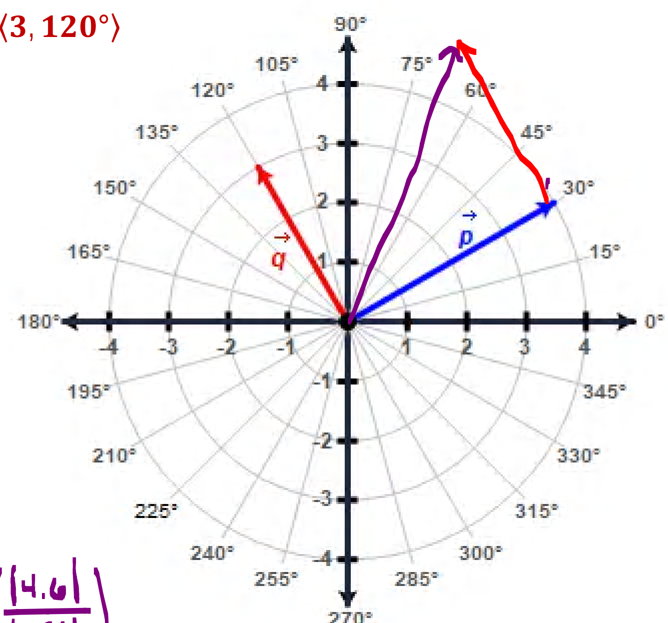
B. Rewrite vector $\vec{q}$ in rectangular form.

$\langle 3\cos 120°, 3\sin 120°\rangle$

3b. $\langle -1.50, 2.60\rangle$

C. Determine $\vec{p} + \vec{q}$ in rectangular form.

3c. $\langle 1.96, 4.60\rangle$

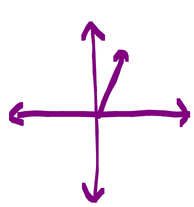


D. Determine $\vec{p} + \vec{q}$ in polar form.

$1.96^2 + 4.60^2 = c^2$

$\sqrt{25} = \sqrt{c^2}$

$5 = c$



$\alpha = \theta = \tan^{-1}\left(\frac{|4.6|}{|1.96|}\right)$

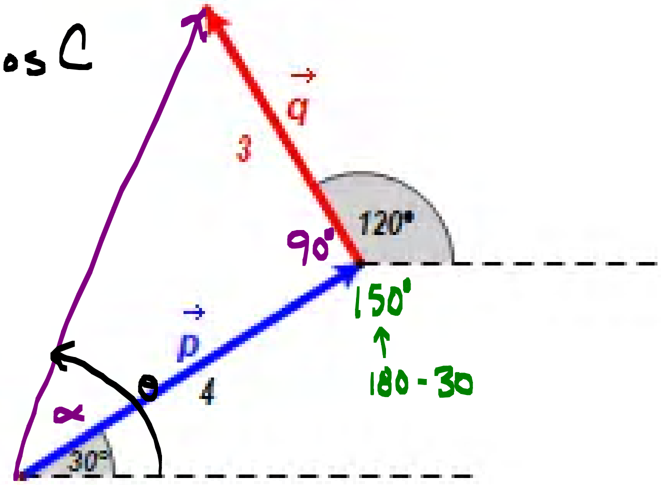
$\approx 66.92°$

3d. $\langle 5, 66.9°\rangle$

4. Let the following vectors be defined: $\vec{p}: \langle 4, 30° \rangle$ $\vec{q}: \langle 3, 120° \rangle$

Determine $\vec{p} + \vec{q}$ in polar form. $c^2 = a^2 + b^2 - 2ab\cos C$

(Use law of Cosines to determine your answer.)



$c^2 = 4^2 + 3^2 - 2(3)(4)\cos(90)$

$\sqrt{c^2} = \sqrt{25}$ $c = 5$

$3^2 = 5^2 + 4^2 - 2(5)(4)\cos\alpha$

$9 = 41 - 40\cos\alpha$

$-41 \quad -41$

$-32 = -40\cos\alpha$

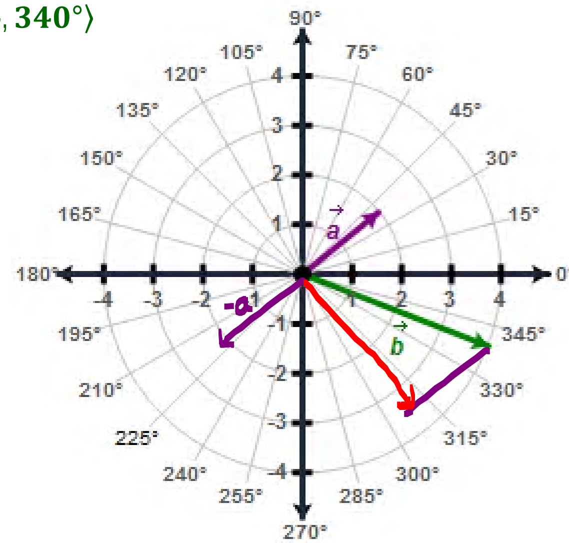
$-40 \quad -40$

$0.8 = \cos\alpha$ $\alpha \approx 36.7$

$\theta \approx 30 + 36.7 \approx 66.7°$

4. $\langle 5, 66.7° \rangle$

5. Let the following vectors be defined: $\vec{a}: \langle 2, 40° \rangle$ $\vec{b}: \langle 4, 340° \rangle$



A. Rewrite vector $\vec{a}$ in rectangular form.

$\langle 2\cos 40, 2\sin 40 \rangle$

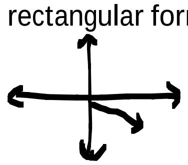
5a. $\langle 1.53, 1.29 \rangle$

B. Rewrite vector $\vec{b}$ in rectangular form.

$\langle 4\cos 340, 4\sin 340 \rangle$

5b. $\langle 3.76, -1.37 \rangle$

C. Determine $\vec{b} - \vec{a}$ in rectangular form.



5c. $\langle 2.23, -2.66 \rangle$

D. Determine $\vec{b} - \vec{a}$ in polar form.

$(2.23)^2 + (-2.66)^2 = c^2$

$\sqrt{12.05} \approx \sqrt{c}$

$3.47 \approx c$

$\alpha = \tan^{-1}\left(\frac{|-2.66|}{|2.23|}\right)$

$\alpha = 50.03°$

$\theta \approx 360° - 50.03$

$\theta \approx 309.97°$

5d. $\langle 3.5, 310.0 \rangle$

E. Determine $5\vec{a}$ in polar form.

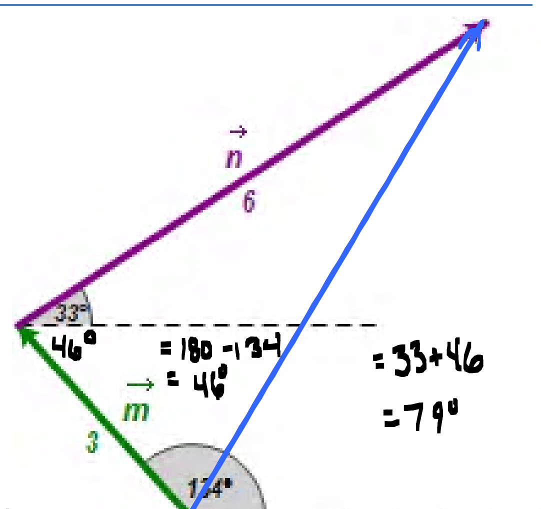
$5 \cdot \langle 2, 40° \rangle$

5e. $\langle 10, 40° \rangle$

6. Let the following vectors be defined: $\vec{m}: \langle 3, 134° \rangle$ $\vec{n}: \langle 6, 33° \rangle$

Determine $\vec{m} + \vec{n}$ in polar form. $c^2 = a^2 + b^2 - 2ab\cos C$

(Use law of Cosines to determine your answer.)



$= 180 - 134$
$= 46°$

$= 33 + 46$
$= 79°$

$c^2 = 3^2 + 6^2 - 2(3)(6)\cos(79)$

$\sqrt{c^2} \approx \sqrt{38.13}$ $c \approx 6.18$

$6^2 = 6.18^2 + 3^2 - 2(6.18)(3)^2\cos\alpha$

$36 = 47.19 - 37.08\cos\alpha$

$-47.19 \quad -47.19$

$-11.19 = -37.08\cos\alpha$

$-37.08 \quad -37.08$

$.3018 \approx \cos\alpha$

$72.4 \approx \alpha$

$\theta = 134 - 72.4 \approx 61.56$

6.

$\langle 6.18, 61.56° \rangle$

7. **Let the following vectors be defined:** $\vec{v}: \langle 6, 38°\rangle$ $\quad \vec{u}: \langle 2, 163°\rangle$

Sketch the vector expression $\vec{v} + 2\vec{u}$ in polar form. Start the vector sketch with the point shown below.

LAW OF COSINES

$c^2 = 6^2 + 4^2 - 2(6)(4)\cos(55)$

$\sqrt{c^2} \approx \sqrt{24.47}$ $\quad c \approx 4.95$

$4^2 = 6^2 + (4.95)^2 - 2(6)(4.95)\cos\alpha$

$16 = 60.47 - 59.36\cos\alpha$

$-60.47 \quad -60.47$

$-44.47 = -59.36\cos\alpha$

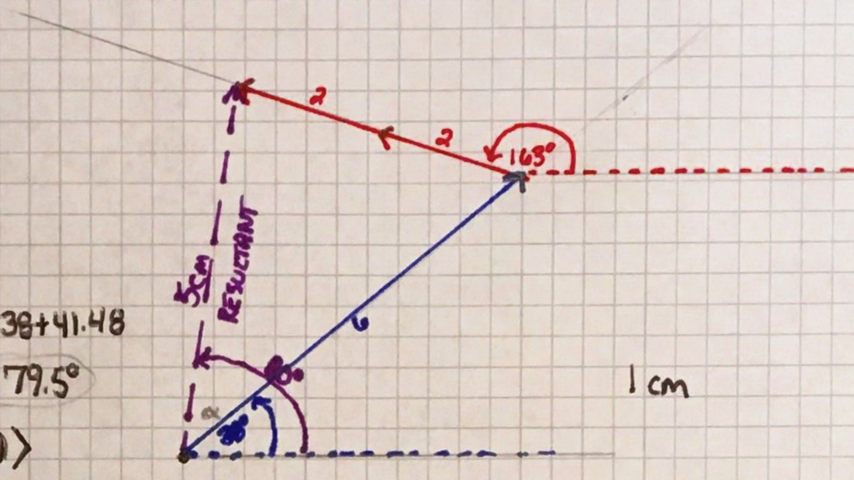
$-59.36 \quad -59.36$

$0.749 \approx \cos\alpha \quad \alpha \approx 41.48 \quad \theta \approx 38 + 41.48$

$\theta \approx 79.5°$

$360 - 142 - 163 = 55°$

$180 - 38$



COMPONENT METHOD

$\vec{v}: \langle 6\cos 38, 6\sin 38\rangle \approx \langle 4.73, 3.69\rangle$

$2\vec{u}: \langle 2\cdot 2\cos 163, 2\cdot 2\sin 163\rangle \approx + \langle -3.83, 1.17\rangle$

$\langle 0.9, 4.86\rangle$

$\langle 4.9, 79.5\rangle$

CONVERT TO POLAR

$c^2 = (0.9)^2 + (4.86)^2$

$\sqrt{c^2} = \sqrt{24.43} \quad c = 4.9$

$\theta = \tan^{-1}\left(\frac{4.86}{.9}\right) \approx 79.5°$

8. Sketch a vector graph of two movers trying to move a washing machine up a set of stairs. The first mover is pushing at an angle of 165° and a force of 140 pounds. The second mover is pulling at an angle of 105° and a force of 130 pounds. What is the estimated magnitude and direction of the resultant force based on your drawing?

LAW OF COSINE METHOD

$c^2 = 140^2 + 130^2 - 2(140)(130)\cos(120)$

$c^2 = 36500 - 36400\cos(120) = 54700$

$c = \sqrt{54700} \approx 233.9\text{ lbs}$

$130^2 = (233.9)^2 + (140)^2 - 2(233.9)(140)\cos\alpha$

$16900 = 74299.85 - 65492\cos\alpha$

$-74299.85 \quad -74299.85$

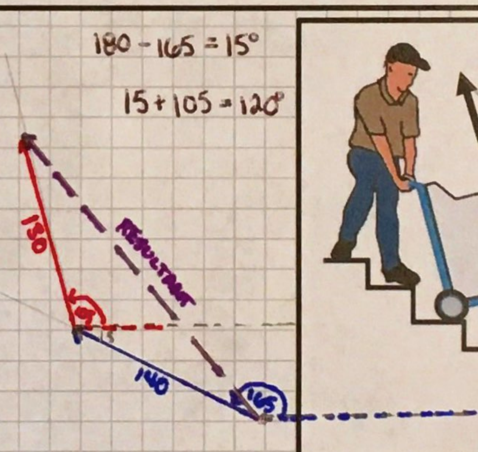
$-57399.85 = -65492\cos\alpha$

$-65492 \quad -65492$

$0.8764 = \cos\alpha \quad \alpha \approx 28.8 \quad \theta \approx 165 - 28.8 = 136.2°$

$180 - 165 = 15°$

$15 + 105 = 120°$



COMPONENT METHOD

$\vec{a}: \langle 140\cos 165, 140\sin 165\rangle \approx \langle -135.2, 36.2\rangle$

$\vec{b}: \langle 130\cos 105, 130\sin 105\rangle \approx + \langle -33.6, 125.6\rangle$

$\langle -168.8, 161.8\rangle$

CONVERT TO POLAR

$c^2 = (-168.8)^2 + (161.8)^2$

$\sqrt{c^2} = \sqrt{54672.68} \approx 233.8$

$\alpha = \tan^{-1}\left(\frac{161.8}{168.8}\right) = 43.8°$

$\theta \approx 180 - 43.8 = 136.2°$

If the movers require a minimum of 200 pounds of force at roughly a 135° angle, do you think they will have enough force to move the washing machine up the stairs?

RESULTANT: ⟨234, 136°⟩ $\quad$ YES

9. A boy is swimming across a river. The boy's path makes a 36° angle with the river bank and the boy is swimming slightly upstream (against the current). The boy is swimming at a rate of 4 feet per second and the current is flowing at 2 feet per second. How fast is the boy actually moving and in what direction?

LAW OF COSINES

$c^2 = (2)^2 + (4)^2 - 2(2)(4)\cos(36°)$

$\sqrt{c^2} \approx \sqrt{7.056} \approx \underline{2.65 \text{ MPH}}$

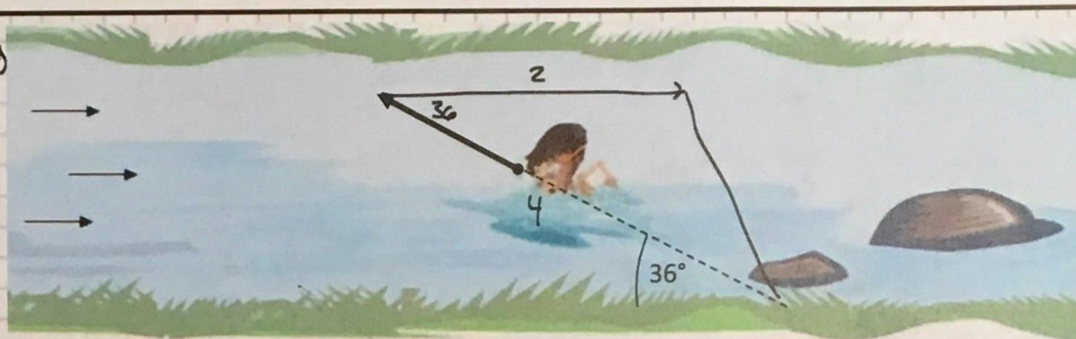
$(2)^2 \approx (2.65)^2 + (4)^2 - 2(2.65)(4)\cos\alpha$

$4 \approx 23.\cancel{06} - 21.25\cos\alpha$

$-23.06 \quad -\cancel{23.06}$

$\dfrac{-19.06}{-21.25} \approx \dfrac{-21.25\cos\alpha}{-21.25}$

$0.8967 \approx \cos\alpha \qquad \alpha \approx 26.3° \qquad \theta \approx 144° - 26.3° \approx \underline{117.7°}$



$180° - 36° = 144°$

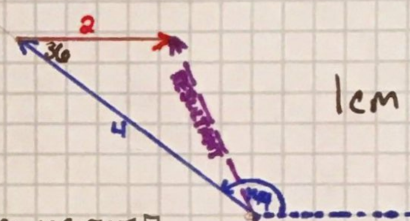
COMPONENT METHOD

$\vec{a} : \langle 4\cos 144°, 4\sin 144° \rangle : \langle -3.24, 2.35 \rangle \qquad c^2 = (-1.24)^2 + (2.35)^2$

$\vec{b} : \langle 2\cos 0°, 2\sin 0° \rangle :_+ \langle 2, 0 \rangle \qquad \sqrt{c^2} = \sqrt{7.056} \approx \underline{2.66 \text{ MPH}}$

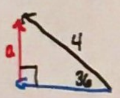
$\langle -1.24, 2.35 \rangle \qquad \tan\alpha = \left(\dfrac{2.35}{1.24}\right)$

$\alpha \approx 62.3 \qquad \theta = 180 - 62.3 \approx \underline{117.7}$



1 cm = 1 MPH

If the river is roughly 31 feet across in width, how many seconds will it take the boy to get across the river?



$\sin 36 = \dfrac{a}{2}$

$a = 4 \cdot \sin 36$

$a \approx 2.35 \text{ MPH}$

$d = rt$

$\dfrac{31}{2.35} = \dfrac{(2.35)t}{2.35}$

$13.19 = t$

$13.2 \text{ sec} = t$

10. Two tow trucks are trying to pull a car out of the mud at the same time. The first tow truck is pulling the car due East with a force of 900 Newtons. The second truck is pulling the same car 34° North of East from the first tow truck with a force of 1400 Newtons. Which direction will the car most likely move and how many Newtons is being applied to the car?

COMPONENT METHOD

$\vec{a} : \langle 900\cos 0°, 900\sin 0° \rangle = \langle 900, 0 \rangle$

$\vec{b} : \langle 1400\cos 34°, 1400\sin 34° \rangle \approx_+ \langle 1160.65, 782.87 \rangle$

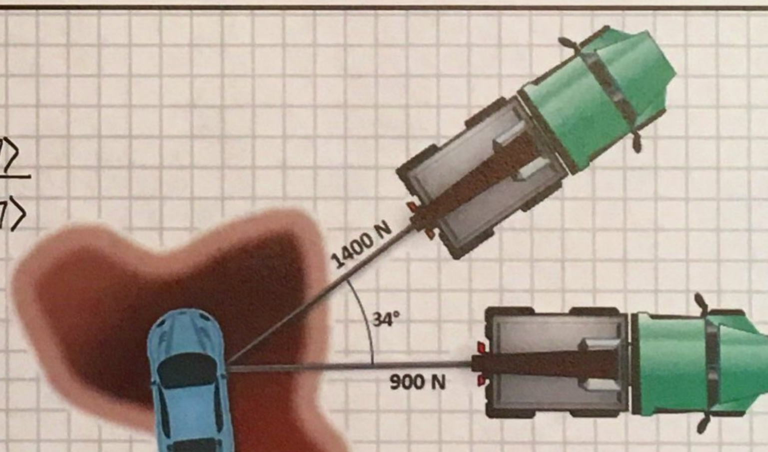
$\langle 2060.65, 782.87 \rangle$

CONVERT TO POLAR FORM

$c^2 = (2060.65)^2 + (782.87)^2$

$\sqrt{c^2} = \sqrt{4859163.859} \approx \underline{2204.35} \text{ NEWTONS}$

$\tan\theta = \left(\dfrac{782.87}{2060.65}\right), \quad \underline{\theta \approx 20.8°}$



LAW OF COSINES

$c^2 = (1400)^2 + (900)^2 - 2(1400)(900)\cos(146°)$

$180° - 34° = 146°$

$\sqrt{c^2} \approx \sqrt{4859174.68} \approx \underline{2204.35 \text{ NEWTONS}}$

1 cm = 300N

$900^2 = (1400)^2 + (2204.35)^2 - 2(1400)(2204.35)\cos\alpha$

$810000 = 6819174.68 - 6172190.01\cos\alpha$

$-6819174.68 \quad -6819174.68$

$\dfrac{-6009174.68}{-6172190.01} = \dfrac{-\cancel{6172190.01}\cos\alpha}{-\cancel{6172190.01}}$

$0.97359 \approx \cos\alpha \qquad \alpha \approx 13.2°$

$\theta \approx 34° - 13.2° \approx 20.8°$